



Numeracy Development Framework

Pathway to College and Career Readiness Through Numeracy Development

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Carey M. Wright, Ed.D.
State Superintendent of Schools

Tenette Smith, Ed. D.
Chief Academic Officer
Office of Teaching and Learning

Wes Moore
Governor

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The Maryland State Department of Education contributors include:

OFFICE OF TEACHING AND LEARNING

Lyndsey Brightful, Director of Mathematics

Elise Brown, Ph.D., Assistant State Superintendent of Instructional Programs

Stacie Marvin, Coordinator of Mathematics

Jen Meehan, Coordinator of Mathematics

Robert Richardson, Ed.D., Coordinator of Mathematics

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The Numeracy Development Framework

Numeracy is more than the ability to carry out mathematical procedures. It involves understanding, reasoning with, and applying mathematical ideas across a range of contexts. Numeracy develops over time as students build knowledge, strengthen reasoning skills, and grow dispositions that support mathematical thinking. This includes the ability to represent and analyze problems, make sense of quantitative relationships, and select and use appropriate mathematical tools to support their thinking.

Mathematical processes (i.e., modeling, using tools and representations, reasoning and proving, and seeing and generalizing structure) are essential throughout the entire trajectory of numeracy development and within each domain outlined below. These mathematical processes, along with productive mathematical habits of mind, underpin students’ ability to apply mathematics in meaningful, creative, and contextually relevant ways as they progress toward college and career readiness.

A student’s relationship with mathematics is shaped not only by what they learn but by how they learn it. Developing a strong mathematical identity and positive disposition requires ongoing opportunities to model with mathematics, collaborate with others, explore multiple strategies and representations, and reflect on their own learning.

This framework focuses on the core grade-level competencies that form the foundation for numeracy development from PreK through Grade 8 to support algebra readiness and access to college and career mathematics opportunities. While these competencies are essential, they represent only one dimension of what it means to be numerate. **The practices, dispositions, and aspects of identity referenced above remain critical and must be actively nurtured alongside skill development. Interpreting and applying these competencies within rich learning environments is key to supporting the full development of numeracy over time.**

The framework is organized into four interconnected domains:

Number & Operation Sense	Algebraic Thinking	Geometric Reasoning & Measurement	Reasoning with Data, Statistics, & Probability
Outlines key milestones in students’ development of understanding and fluency with number across four interconnected strands: Numbers & Number Sets, Operations, Estimation, and Fractions & Decimals.	Outlines key milestones in the development of students’ ability to represent and reason about mathematical relationships across four interconnected strands: Creating Equations & Inequalities, Solving Equations & Inequalities, Functional Reasoning, and Proportional Reasoning.	Outlines key milestones in students’ ability to analyze, describe, and work with shapes, spatial relationships, and measurement across four interconnected strands: Classifying Figures, Measurement, Reasoning with Geometric Structure, and Spatial Reasoning.	Outlines key milestones in students’ ability to collect, represent, analyze, and make sense of data across three interconnected strands: Collecting, Representing, & Describing Data, Interpreting & Analyzing Data, and Reasoning about Probability.

Each domain reflects a developmental progression from fundamental concepts in early grades to the essential understandings and skills needed for readiness in the Integrated Algebra sequence and beyond, and provides students with:

- A multidimensional understanding of numerical and algebraic relationships
- Spatial awareness and geometric modeling capabilities
- Data literacy and critical statistical reasoning skills.

The mathematical processes described above are not separate from content. They underlie how students engage with the milestone competencies in each domain of this framework. These practices (i.e., modeling, using tools and representations, reasoning and proving, seeing and generalizing structure, and cultivating mathematical habits of mind) support students in building conceptual understanding, making connections across mathematical ideas, and applying mathematics in flexible and purposeful ways. Their integration across the numeracy domains reinforces the coherence of the framework and supports students in becoming confident, capable problem-solvers prepared to meet the mathematical demands of college, career, and life.

NUMBER & OPERATION SENSE

Across all grades, when working with numbers (including whole numbers, integers, decimals, fractions, rational and irrational numbers), **students are expected to:**

- Demonstrate progressively flexible number sense through the use of multiple representations, including concrete (e.g., physical manipulatives), semi-concrete (e.g., drawings, number bonds, number lines, tape diagrams), and abstract models (e.g., numerals, symbols), to reason about and operate with whole numbers, integers, decimals, fractions, and rational numbers.
- Use place value understanding to compare, compose, and decompose whole numbers, and eventually decimals, based on the meanings of ones, tens, hundreds, and beyond.
- Use precise and appropriate mathematical vocabulary when describing and reasoning about numbers, operations, and their relationships¹.
- Consistently explain why solutions and chosen procedures make sense. Proficiency with operations necessarily includes conceptual understanding of the mathematics involved, not just getting correct answers.

	By the end of PreK, I can...	By the end of K, I can...	By the end of 1 st grade, I can...	By the end of 2 nd grade, I can...	By the end of 3 rd grade, I can...	By the end of 4 th grade, I can...	By the end of 5 th grade, I can...	By the end of 6 th grade, I can...	By the end of 7 th grade, I can...	By the end of 8 th grade, I can...
Numbers & Number Sets	• Recognize quantities of up to 3-5 objects in familiar arrangements without counting². • Count and compare quantities up to 10.	• Recognize quantities of up to 10 without counting by visually identifying and combining smaller, familiar groups³. • Read, write, compose, decompose, and compare numbers through 20, using multiple representations.	Read, write, compose, decompose, and compare numbers through 100, using multiple representations.	Read, write, compose, decompose, and compare numbers through 1,000, using multiple representations.	• Read, write, compose, decompose, and compare numbers through 10,000, using multiple representations. • Read, write, and communicate about fractions as equal parts of a whole.	• Read, write, compose, decompose, and compare numbers through 1,000,000, using multiple representations. • Read, write, and compare decimals to the hundredths place.	Read, write, compose, decompose, and compare numbers through millions and thousandths, using multiple representations.	• Represent results of division as fractions, decimals, and percents. • Represent positive and negative numbers on a number line and interpret in contexts⁴.		Use scientific notation to express very large and very small numbers.
Operations	Identify the number that is one more or one less than a given number by adding or removing from a set of objects of up to 10 items.	Use counting to solve addition and subtraction problems within 10.	Use and compare various strategies to solve addition and subtraction problems within 20.	• Recall or quickly derive single-digit addition and related subtraction facts. • Fluently⁵ solve addition and subtraction problems within 100.	• Recall or quickly derive all products of two one-digit numbers. • Use properties of operations⁶ to solve multiplication / division problems.	• Fluently solve addition and subtraction problems within 1,000,000. • Use reasoning-based strategies for multiplication / division with multi-digit whole numbers⁷.	• Fluently multiply multi-digit whole numbers. • Use reasoning-based strategies⁸ for multi-digit division.	• Fluently divide multi-digit whole numbers. • Use absolute value to describe magnitude, distance, or difference between numbers.	Fluently add, subtract, multiply, and divide positive and negative rational numbers.	Apply properties of exponents to generate equivalent numerical expressions.
Estimation			Use benchmark numbers (e.g., 0, 50, 100) to reason about quantities, comparisons, and relationships.	Determine when using estimation is appropriate ⁹ .	Use estimation strategies to assess the reasonableness of sums and differences involving large numbers.	Use estimation strategies to assess the reasonableness of sums, differences, and products involving large numbers.	Use estimation strategies to assess the reasonableness of sum, difference, products, and quotients involving large numbers.	Use estimation strategies to assess the reasonableness of answers involving fraction and decimal operations.	Use estimation strategies to assess the reasonableness of answers involving positive and negative rational numbers.	Use rational approximations of irrational numbers to estimate the value of expressions involving irrational numbers ¹⁰ .
Fractions & Decimals			Partition rectangles and circles into halves and fourths.	Partition rectangles and circles into halves, fourths, and thirds.	• Represent and compare fractions by counting unit fractions, using area and length models, and benchmark reasoning. • Show that the size of a fractional part depends on the size of the whole¹¹.	• Generate equivalent fractions. • Fluently add/subtract fractions with like denominators.	• Fluently add/subtract fractions with unlike denominators. • Multiply a fraction or whole number by a fraction.	• Fluently multiply / divide fractions. • Fluently add/subtract multiply/divide decimals.	Fluently convert between rational numbers and percents.	

¹ This includes both verbal and written representations, such as using clear and precise fraction notation, including a horizontal vinculum to show the structure of numerators and denominators.

² This is called [perceptual subitizing](#).

³ For example, seeing an arrangement of 8 dots as a group of 5 and a group of 3 in a familiar arrangement. This is called [conceptual subitizing](#).

⁴ For example, temperature change, financial transactions, elevations below sea level.

⁵ *Fluently*, in Number & Operation Sense, means computing with accuracy, efficiency, and flexibility, guided by the reasonableness of the strategy selected, how it is used, and the resulting solution.

⁶ Commutative, associative, and distributive properties. For example, to solve 8 × 7, think of (5 × 7) + (3 × 7)) to multiply using known facts.

⁷ Using place value understanding and properties of operations (especially the distributive property). Also includes using models (e.g., equal group, area models, partial products, and arrays for multi-digit multiplication; and partitioning, successive subtraction, inverse of multiplication for multi-digit division).

⁸ Using place value understanding, properties of operations, and the inverse relationship between multiplication and division.

⁹ Estimating how many second graders in the school vs. giving the exact number of second graders present in class.

¹⁰ For example, use 3.14 to approximate π in calculating an approximate area of a circle.

¹¹ For example, creating visual models, such as partitioned shapes or number lines, to show how equivalent fractions can represent different quantities when applied to wholes of varying sizes.

ALGEBRAIC THINKING

Across all grades, when engaging in algebraic thinking, **students are expected to:**

- Recognize and describe patterns in numbers and operations, and use those patterns to identify relationships and make generalizations.
- Reason about structure in expressions and equations, identify regularity in relationships, and connect specific examples to general rules.

	By the end of PreK, I can...	By the end of K, I can...	By the end of 1 st grade, I can...	By the end of 2 nd grade, I can...	By the end of 3 rd grade, I can...	By the end of 4 th grade, I can...	By the end of 5 th grade, I can...	By then end of 6 th grade, I can...	By the end of 7 th grade, I can...	By the end of 8 th grade, I can...
Creating Equations & Inequalities		Recognize and explain when two groups have the same amount, using objects, drawings, or equations to show equality.	• Explain the meaning of the equal sign in equations. • Represent one-step addition and subtraction situations within 20 using equations with a symbol for the unknown quantity.	Represent one- and two-step addition and subtraction situations within 100 using equations with a symbol for the unknown quantity.	Represent contexts involving multiplication and division using equations with a letter for the unknown quantity.	Represent contexts involving combinations of all four operations using equations with a letter for the unknown quantity ¹² .	Represent contexts using numerical expressions with appropriate grouping symbols to reflect relationships between quantities and ensure correct order of operations.	• Write algebraic expressions using variables to represent relationships described in context or narrative form¹³. • Represent linear algebraic expressions in equivalent forms using properties of operations¹⁴. • Represent contexts using inequalities in the form $x > c$, $x < c$, $x \geq c$, $x \leq c$.	Represent contexts using equations or inequalities, such as $px + q = r$ or $px + q > r$, where p, q, and r are rational ¹⁵ .	Represent contexts involving two related quantities using systems of linear equations.
Solving Equations & Inequalities	Use objects or drawings to determine how many more are needed to make quantities up to 5.	Use objects, drawings, or equations to solve addition and subtraction problems ¹⁶ within 10.	Determine the unknown whole number in an addition or subtraction equation within 20 using a context or situation to make sense of the equation.	Determine the unknown whole number in each step of a two-step addition and/or subtraction problem within 100 by reasoning about and solving equations based on the context ¹⁷ .	Solve for the unknown whole number in a multiplication or division equation ¹⁸ , and check solutions to confirm the equation is true.	Solve for the unknown whole number in a multi-step problem by reasoning about one or more equations, and check solutions using the meaning of the equal sign.		Represent and interpret the solution of an equation and the solution set of an inequality on a number line.	Select and apply efficient approaches to solve linear equations and inequalities based on the structure of the equation/inequality.	Solve systems of two linear equations in two variables by graphing and substitution, and interpret the solution.
Functional Reasoning					Recognize and describe patterns in numbers that reflect a predictable relationship between quantities (e.g., if 1 bicycle has 2 wheels, then 2 bicycles have 4 wheels, 3 have 6, and so on).	Describe and represent multiplicative relationships between two quantities using explicitly defined rules (e.g., The number of wheels is 2 times the number of bicycles; represented by the rule number of wheels = 2 x number of bicycles).	• Use explicitly defined rules to generate input-output pairs. • Represent the relationship as a set of ordered pairs.	Describe how changes in the independent variable are reflected in the dependent variable.	• Explain how a consistent multiplicative rule connects values in a proportional relationship. • Describe what specific points on a graph mean in terms of input/output of the situation.	Determine whether a given relationship is a function.
Proportional Reasoning						Distinguish between additive and multiplicative relationships by analyzing and comparing patterns in context.		Use multiple representations of proportional relationships, including ratio tables, to identify equivalent ratios, determine unknown values within proportional relationships, and find unit rates.	• Distinguish between proportional and non-proportional relationships by determining whether the ratios between quantities represent a constant rate. • Graph proportional relationships and use the graph to interpret and compare unit rates.	• Given a linear relationship, identify the rate of change and initial value from a table, graph, or description, and write a linear function to model the relationship. • Describe how nonlinear relationships differ from linear relationships.

¹² Isabel has 120 minutes of free time. She uses it to play a game that takes 12 minutes per round. If she plays 3 rounds of the game, how much free time will she have left? ($120 - 12 \times 3 = M$ or $12 \times 3 = 36$; $120 - 36 = M$).

¹³ For example, express “subtract y from 5” as $5 - y$ or “twice the sum of a and b” as $2(a + b)$.

¹⁴ For example, representing $2(3x + 4)$ as $6x + 8$ through use of the distributive property

¹⁵ This includes positive and negative integers, fractions, and decimals.

¹⁶ For more information about addition and subtraction situations by grade level (K - Grade 2), [see this resource](#).

¹⁷ For example, A class had 18 markers. After 7 dried out and were thrown away, the teacher bought 5 new ones. How many markers does the class have now? ($18 - 7 = 11$; $11 + 5 = 16$).

¹⁸ For more information about multiplication and division situations by grade level, [see this resource](#).

GEOMETRIC REASONING & MEASUREMENT

Across all grades, when working with geometric reasoning and measurement, students are expected to:

- Recognize shapes in different sizes and orientations and identify attributes that remain the same (e.g., a triangle always has three sides, regardless of its size or how it is turned).
- Develop and apply formulas by building conceptual understanding of the attributes being measured and by informally deriving why the formula works based on the structure of the shape.
- Solve problems in authentic contexts as they classify shapes, reason about spatial relationships, and apply measurement concepts.
- Use precise mathematical vocabulary to describe shapes, spatial relationships, and measurement attributes, and to justify reasoning.

	By the end of PreK, I can...	By the end of K, I can...	By the end of 1 st grade, I can...	By the end of 2 nd grade, I can...	By the end of 3 rd grade, I can...	By the end of 4 th grade, I can...	By the end of 5 th grade, I can...	By the end of 6 th grade, I can...	By the end of 7 th grade, I can...	By the end of 8 th grade, I can...
Classifying Figures	Sort two-dimensional shapes based on observable attributes such as number of sides or corners.	Sort two- and three-dimensional shapes based on their attributes.	Describe categorical attributes ¹⁹ of shapes ²⁰ .		Explain that some shapes belong to more than one category based on their attributes ²¹ .	Classify two-dimensional shapes by parallel and perpendicular lines ²² .	Classify quadrilaterals and triangles based on angle measures and side length.			Use angle relationship classifications (supplementary, complementary, vertical, adjacent) to find unknown angle measures.
Measurement	Sort and compare physical objects by size.	Compare two objects with a measurable attribute in common using phrases like “holds more/less”, “longer/shorter”.	• Estimate and order the lengths of objects using direct and indirect comparisons. • Measure length of physical objects using non-standard units²³.	• Use benchmark measurements to estimate length in standard units (inches, centimeters, meters). • Measure length of physical objects using standard units.	• Use benchmark measurements to estimate liquid volume in standard units (e.g., liters). • Measure liquid volumes in standard units.	• Use benchmark angles (e.g., 90°, 45°) to estimate angle measures. • Measure angles in whole-number degrees with a protractor.	Convert between different-sized units within the same measurement system.			
Reasoning with Geometric Structure		Compose shapes by combining basic shapes ²⁴ to form new ones.	Compose two- and three-dimensional shapes (e.g., rectangles, triangles, prisms, cylinders) to build composite shapes, and describe how composite shapes can be made from or taken apart into other shapes.		Apply understanding of geometric structure to: • Determine perimeter of physical and described objects. • Determine area of rectangles.	• Find missing dimensions of rectangles using knowledge of area and perimeter. • Relate the area of a triangle to the area of a rectangle through decomposition.	Apply understanding of geometric structure to determine volume of rectangular prisms.	• Find the area of polygons by composing or decomposing them into triangles and quadrilaterals. • Use nets to represent three-dimensional figures defined by edges and flat faces and calculate surface area.	Apply understanding of geometric structure to: • Determine the area and circumference of circles. • Determine surface area and volume of cones, cylinders, and spheres.	• Use the Pythagorean Theorem to find unknown side lengths in right triangles. • Find missing angles in triangles by knowing that the sum of the interior angles is always 180°.
Spatial Reasoning		Describe the location of objects using positional words such as above, below, beside, in front of, behind, and between.		Identify line-symmetric figures and draw lines of symmetry.			Use a coordinate system with perpendicular number lines to locate points using ordered pairs, with both values positive.	Draw polygons in the coordinate plane and determine side length by counting units between points with the same x- or y-coordinate.	Describe the two-dimensional shapes formed by slicing three-dimensional figures with a plane, and explain how those cross-sections relate to the structure of the solid ²⁵ .	Determine whether a triangle is formed from a given set of three angles or sides.

¹⁹ Categorical attributes are not measurable but serve as defining characteristics for recognizing and classifying shapes. For example, a triangle has three sides, a cube has six square faces, etc.

²⁰ Circle, triangle, rectangle, square, hexagon, cone, cube, cylinder, and sphere

²¹ For example, a square belongs to the categories of rectangles, rhombuses, and quadrilaterals.

²² For example, a rectangle has two pairs of parallel lines and four right angles formed by perpendicular lines.

²³ For example, use paper clips placed end to end with no gaps or overlaps to measure the length of a pencil to the nearest whole paper clip.

²⁴ *Basic shapes* include circles, squares, triangles, and rectangles.

²⁵ For example, slicing a right circular cylinder vertically through its center results in a rectangle, revealing the height and diameter. A horizontal slice results in a circle, matching the base. Together, these cross-sections show that the cylinder has a constant circular cross-section along its height.

REASONING WITH DATA, STATISTICS, & PROBABILITY

Across all grades, when working with data, **students are expected to:**

- Pose statistical questions and collect, represent, and describe data to explore patterns and variability.
- Recognize and reason about uncertainty and likelihood in familiar contexts.
- Critically analyze and interpret data representations and claims by considering how choices in data collection, visualization, and analysis can influence interpretation.
- Describe how data can be used to represent or misrepresent individuals and communities, and develop an awareness of how data reflects power, bias, and context..

	By the end of PreK, I can...	By the end of K, I can...	By the end of 1 st grade, I can...	By the end of 2 nd grade, I can...	By the end of 3 rd grade, I can...	By the end of 4 th grade, I can...	By the end of 5 th grade, I can...	By the end of 6 th grade, I can...	By the end of 7 th grade, I can...	By the end of 8 th grade, I can...
Collecting & Representing Data	Sort up to 10 objects into categories using physical or pictorial representations.	Sort up to 20 objects into categories using physical or pictorial representations.	• Pose questions that can be answered by collecting categorical data. • Represent collected categorical data with tally charts.	• Pose questions that can be answered by collecting numerical or categorical data. • Represent categorical data with bar graphs. • Represent numerical data using line plots.	Determine an appropriate representation to display and interpret data based on the type of data collected.	Explain how changes in scale, order, or type of data visualization affect the story the data tells.	Evaluate whether a data visualization accurately represents the data it is intended to represent.	• Differentiate between statistical and non-statistical questions. • Represent numerical data with histograms and box plots.	• Represent a population by generating a random sample. • Determine whether a sample is representative of a population.	• Represent bivariate measurement data with scatter plots. • Represent data on two categorical variables from the same subjects with two-way tables.
Interpreting & Analyzing Data		Order categories by count.	Answer questions ²⁶ about categorical data sets from tally charts.	Answer questions about numerical data represented in line plots.	• Use patterns in data to make generalizations about the context. • Evaluate whether generalizations about the context are reasonable.		Interpret data using mode, range, and gaps ²⁷ in the distribution.	Use measures of center (median, mean) and variability (interquartile range) to describe and compare data distributions.	Use data visualizations and summary statistics (measures of center, variability) from random samples to compare populations.	Use linear models to analyze associations between variables and make predictions.
Reasoning about Probability							Determine whether a general statement about what usually happens or how often something occurs is reasonable based on the data and the context.	Use probability terms (e.g., likely, unlikely, certain) to describe the likelihood of outcomes and anticipate which responses may be more or less common in data.	• Estimate probabilities of chance events using the context or long-run relative frequency from a simulation. • Use representations (e.g., organized lists, tables, or visual models) to show the sample space and determine the theoretical probability of a simple event with equally likely outcomes.	• Model the sample space for a compound event. • Determine the probability of a compound event.

²⁶ For example, what category had the most data? The least data?

²⁷ For example, students collected the number of books classmates read over the summer, organized in a line plot, and discovered there was a gap between 6 and 9. This is interpreted as no one read 7 or 8 books.