



Grade 3

Maryland College and Career Ready Standards for Mathematics

Companion Guide

Mathematics Branch

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Purpose

PURPOSE OF COMPANION GUIDE

This document is designed to support a shared understanding of what each mathematics standard means, how it connects to other content, and how it can be authentically enacted in classrooms. It does not replace your high-quality instructional materials (HQIM) but instead works alongside them so that educators have a common interpretation of the mathematical expectations we set for students. Regardless of the HQIM in use, these documents provide clarity about the essence and intentions of the standards and illustrate how those expectations can be realized through instruction.

DOCUMENT STRUCTURE

Each domain begins with a set of cluster connections to the Numeracy Development Framework (NDF). These connections identify the ways in which individual clusters either:

- **Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.
- **Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

For each content standard, the companion document provides:

- **Content Progression:** A description of the content that comes before and after that standard to highlight how it fits within a coherent learning trajectory.
- **Examples and Clarifications:** Explanations and expansions of the standard language that provide additional detail about its intent. These may include illustrative examples, as well as common missteps to avoid that can narrow or distort the mathematical authenticity of the standard.
- **Student Opportunities for Mathematical Experience:** Statements describing the kinds of reasoning, modeling, problem solving, and sense-making students should have the chance to engage in as part of learning the content of the standard. Where relevant, these are connected to the Standards for Mathematical Practice (SMPs). Not every SMP is explicitly called out, but all can be present in any standard depending on how instruction is designed. What is included are non-exhaustive examples.

Standards for Mathematical Practice

“The Standards for Mathematical Practice (SMPs) describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” ([Common Core State Standards Initiative](#)). The SMPs develop along a continuum as students progress mathematically, with each grade’s content providing distinct opportunities for students to engage in and demonstrate the practices at an appropriate level of sophistication.

The SMPs should be integrated with content and not taught separately. As students participate in these practices on a regular basis, they become part of the mathematically productive habits of mind that support deep understanding. Intentionally incorporating opportunities to develop the SMPs enhances instruction and enriches students’ mathematical engagement. Students who learn to use these eight mathematical practices as they engage with mathematical concepts and skills have a greater chance of developing conceptual understanding (Van de Walle, et al, 2014).

The table below highlights opportunities to develop the SMPs in general across the given grade band, while the companion guides for individual standards provide more specific connections to the content of each grade. Educators should utilize this resource to assist in planning for opportunities when the SMPs can be emphasized in daily instructional practice. To facilitate the incorporation of SMPs within instruction, notations have been made in the Companion Guide to indicate when a specific SMP may be applicable for a particular standard. These examples should not limit educators from making additional meaningful connections to SMPs where appropriate.

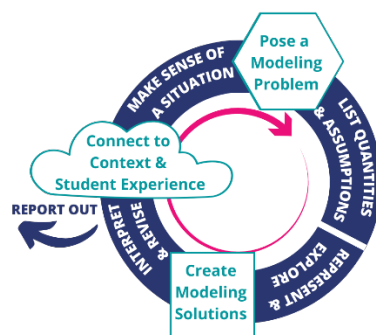
Standard for Mathematical Practice	What does it look like for Grades 3 – 5 students?	How can Grades 3 – 5 teachers support this practice?
Make sense of problems and persevere in solving them (SMP 1)	Students make sense of problems such as multi-step word problems, equal sharing, or comparing quantities using operations, area models, fraction strips, or other visual models. They identify what is being asked, plan an approach (e.g., draw a diagram, make a table), and track if it’s working. They revise when needed and use mathematical reasoning to determine if an answer is reasonable (e.g., “If I’m sharing 12 among 4 people, each should get more than 2”). Students increasingly view perseverance and revising strategies as part of doing math.	Teachers provide contextual problems aligned to grade-level content (operations, fractions, measurement) and build in time for students to plan their approach, often beginning with estimation or sense making. Students are encouraged to consider multiple solution paths and representations. When students get stuck, teachers prompt re-engagement (“What’s another way to show this?”) rather than redirect to correctness. Recognition is given not only for the correct answer but for strategic effort.

Standard for Mathematical Practice	What does it look like for Grades 3 – 5 students?	How can Grades 3 – 5 teachers support this practice?
Reason abstractly and quantitatively (SMP 2)	Students interpret problem situations using numbers, diagrams, and equations and attend to what each quantity represents (e.g., “12 is how many students, not how many buses”). They reason about relationships (e.g., understanding 3×4 as 3 groups of 4) and check whether representations make sense in context. Increasingly, students choose representations that help them reason about the situation, not just carry out a calculation.	Teachers offer opportunities for students to select appropriate representations (e.g., arrays, tables, diagrams, equations) and ask “What does this number or step tell us in the situation?” They facilitate flexible movement between context and representation, and emphasize that understanding quantities and their meaning supports effective reasoning before, during, and after calculation.
Construct viable arguments and critique the reasoning of others (SMP 3)	Students explain how and why their strategy works using models, numbers, words, or equations (e.g., “I multiplied because there were three equal groups”). They listen to other’s explanations, compare approaches, and ask clarifying questions to understand someone else’s reasoning. They begin to revise their own reasoning when a peer’s explanation reveals a clearer, more accurate, or more general way to make sense of the problem.	Teachers create structured opportunities for students to share and compare strategies and use prompts such as, “Can you explain why this works?” or “What makes this approach make sense?” They establish norms that emphasize reasoning and justification as tools for understanding and encourage students to listen to, respond to, and build on one another’s explanations in respectful, constructive ways.
Model with mathematics (SMP 4)	Students ask questions that can be explored mathematically (“How many do we need?”, “How long will it take?”, “What’s the change?”) and identify the key quantities in the situation that need to be represented. They form a mathematical representation (e.g., diagram, table, number line, graph, or equation) to analyze what is happening. They interpret what their model shows and adjust the model when something doesn’t fit or when another choice provides clearer insight. Their modeling cycle becomes define quantities → represent → analyze → interpret → adjust in an increasingly mindful way.	Teachers design tasks where mathematics helps illuminate a real or mathematical situation and prompt students to identify important quantities (“What do we need to keep track of?”). They support students in forming models that match the situation and encourage checking for alignment (“Does your model fit the scenario?”). Multiple models may be compared, but the emphasis stays on how the model improves understanding, not on producing a variety of representations.
Use appropriate tools strategically (SMP 5)	Students select tools, including diagrams, rulers, protractors, number lines, area models, tables, graphs, and digital tools, to help investigate situations or support reasoning. They begin choosing tools purposefully (“A table helps me see the pattern”), not just because the tool is familiar. They recognize when a chosen tool isn’t giving the insight they need and switch to another that better fits the task.	Teachers introduce a variety of tools and help students discover what each tool can reveal about a situation. They ask reflective questions such as “Which tool could help you see this more clearly?” or “What does this representation show that another does not?” Teachers encourage evaluating the strengths and limitations of different tools and ensure tools are used to support, not replace, mathematical reasoning.

Standard for Mathematical Practice	What does it look like for Grades 3 – 5 students?	How can Grades 3 – 5 teachers support this practice?
Attend to precision (SMP 6)	Students use increasingly accurate mathematical language, notation, and units to communicate their thinking. They label representations (e.g., diagrams, number lines, tables, bar graphs, and line plots) to show what quantities or categories mean. They check calculations for accuracy and reasonableness in context, including in data displays (“Does this point belong here?”). Students notice when imprecision in language, diagrams, labels, or data representations leads to confusion and revise their work to make their thinking clearer.	Teachers emphasize naming quantities, labeling representations (including axes and data categories), choosing appropriate scales, and using correct units. They prompt students to clarify what numbers, labels, and symbols represent (e.g., “ <i>What does this number describe?</i> ” or “ <i>What does this label tell the reader?</i> ”) and support students in checking that their representations and calculations are clear and accurate. Precision is reinforced as a habit of communicating mathematical ideas so others can understand them.
Look for and make use of structure (SMP 7)	Students use structure to understand operations more deeply. For example, seeing multiplication as equal groups or arrays, understanding division as sharing or grouping, and using place-value structure to support efficient reasoning. They apply structure to decompose numbers, expressions, and shapes (e.g., “ 24×6 is like $(20 \times 6) + (4 \times 6)$ ”), and they distinguish between additive and multiplicative situations. Students recognize when problems share a common structure and use that insight to choose strategies and explain why those strategies work.	Teachers emphasize structural relationships in arithmetic (arrays, area models, place-value relationships, and properties of operations). They guide students in articulating how the structure of numbers, shapes, or expressions supports a strategy and prompt them to contrast structurally different problem types. Representations are used to make underlying structure visible and to reinforce conceptual understanding rather than procedural repetition.
Look for and express regularity in repeated reasoning (SMP 8)	Students observe regularity in operations and repeated calculations. For example, noticing how products grow in predictable ways when one factor increases, or how repeated addition produces equal groups and eventually multiplication. They recognize stable place-value relationships (“Each shift to the left makes the value ten times larger”) and see how similar reasoning approaches reappear across different division or fraction problems. Students use these repeated behaviors to predict outcomes, check their work, or form generalizations about how operations behave.	Teachers highlight repeated reasoning in multiplicative situations, place-value patterns, fraction equivalence, and strategies that rely on predictable changes (e.g., adding another equal group or repeatedly partitioning a shape or set). They prompt students to describe what repeats across cases (“What changes in the number each time you shift one place to the left?”) and use that insight to make or test conjectures. Teachers encourage students to explain <i>why</i> a shortcut or generalization holds by connecting it to the repeated reasoning that produced it.

Modeling

Mathematical modeling is the process of using mathematics to interpret and make sense of the world. Developing and engaging with the modeling cycle is a critical part of building students' mathematical proficiency. As students progress through the grades, they take on greater responsibility in all aspects of the modeling cycle. They more independently formulate questions, define constraints, and distinguish between assumptions that can be made and facts that must be verified. They are provided growing opportunities to choose models and tools that help them explore a situation and uncover the underlying mathematics. They also become more adept at evaluating the validity of their solutions and revisiting earlier stages of the cycle when initial assumptions prove invalid or incomplete. Participation in the mathematical modeling cycle nurtures mathematical creativity, fosters productive discourse, encourages persistence when solutions are not straightforward, and supports the development of positive mathematics identity.



Mathematical modeling is a significant aspect of MCAP Grades 3-8. Modeling is assessed in alignment to grade-level content. Modeling items are both machine scored and constructed response. The constructed response items are scored using [MCAP Mathematics Modeling Holistic Rubric](#). A calculator will be available for all Modeling assessment items.

Opportunities to connect modeling with content standards are identified within the Companion Guide. These opportunities are entry points for integration, but they should not limit educators from making additional meaningful connections to modeling where appropriate.

Modeling Evidence Statements	What does this look like for students in Grades 3 – 5?
M.1 Formulate a question about a particular context that can be answered using mathematics.	Students examine real or imagined contexts and formulate questions that can be answered using mathematics. They move beyond general curiosity to identify specific quantities, comparisons, or relationships that can be measured, counted, or calculated. For example, given a situation about planning a school fundraiser, students might refine a broad question such as “Will we raise enough money?” into questions like “How much money will we raise if each student sells three items?” or “How many items need to be sold to reach the fundraising goal of \$500?”
M.2 Determine the information needed and what assumptions must be made to answer a mathematical question about a context.	Students examine a situation to decide what details matter when answering a mathematical question. They sort relevant from irrelevant information, identify what is already known and what still needs to be determined, and recognize when assumptions are necessary to proceed. At this grade band, assumptions are typically based on simplifying the context so it can be analyzed mathematically. For example, in a school fundraiser scenario, students may decide they need to know how many students are participating and the price of each item sold. They may assume that all items cost the same or that each student sells the same number of items to estimate the total amount raised.
M.3 Represent a context mathematically by selecting and applying appropriate concepts, tools, and representations to define variables, model relationships, and/or identify constraints.	Students organize information from a situation to show how quantities are connected. They use representations such as drawings, tables, diagrams, equations, and simple graphs to make relationships visible and support their thinking. As they work, students attend to limits or conditions in the situation that affect how the context can be represented. For example, when modeling a school fundraiser, students may create a table showing how many items are sold and the total money raised or write a relationship such as <i>total money = number of items × price</i> to represent how the quantities are related.
M.4 Use a mathematical model to perform computations, make predictions, or answer the question posed about a context.	Students engage with their models as tools for exploring how quantities in a situation are related and to support decisions/conclusions about the context. They use the model to compare scenarios, notice patterns, and test how changes to one part of the situation affects others. Their models are used to guide their thinking rather than used for a single calculation. For example, using a fundraiser model, students may explore how increasing the number of items sold changes the total amount raised and use that relationship to compare different sales scenarios or consider how adjusting the price per item would affect the total.

Modeling Evidence Statements	What does this look like for students in Grades 3 – 5?
M.5 Interpret the results or conclusions of a mathematical model in terms of the original context or situation.	Students reflect on what their model reveals about the situation and connect the outcomes of the model back to the original question. They explain what the numbers or comparisons produced by the model mean in context and what those results suggest about decisions or next steps. For example, after modeling a school fundraiser, students may explain whether the amount of money raised meets the goal, describe what comparisons between sales plans show, or explain what the model suggests about how changing price affects outcomes.
M.6 Critique the validity of a mathematical model and its conclusions. Revise assumptions, representations, or mathematical approaches based on the context, new understanding, or additional information.	Students examine whether a mathematical model and its conclusions make sense for the situation being studied. Through sharing their thinking and comparing models with peers, students consider whether assumptions were reasonable, whether important details were overlooked, and whether the model reflects realistic constraints of the context. Based on new understanding, feedback, or additional information, students revise their assumptions or adjust their model to better represent the situation. For example, after sharing fundraising models, students may notice that another group included a limit on how many items a student could reasonably sell. Recognizing that this constraint changes the total amount raised, they revise their assumptions and adjust the model before reconsidering their conclusions.

Reasoning

Students who become adept at reasoning in mathematics are better able to transfer their learning to new and novel contexts and build confidence in their mathematical abilities. Reasoning is both a foundational skill for students and a critical focus of instruction for mathematics educators. Opportunities to reason should be embedded throughout all grade levels and students must be invited to justify their solutions, use multiple representations and tools to support conclusions, and engage in meaningful reflection. They should frequently be called upon to evaluate their own reasoning and that of others, decide whether a solution path is valid, identify and correct errors, and strengthen arguments. As outlined below, mathematical reasoning is a cornerstone skill that should be integrated across all grade-level content.

Reasoning is a significant topic in the MCAP assessment. Reasoning is assessed in alignment to grade-level content. Reasoning items are both machine scored and constructed response. The constructed response items are scored using [MCAP Mathematics Reasoning Holistic Rubric](#). A calculator will be available for all Reasoning assessment items.

Opportunities to connect reasoning with content standards are identified within the Companion Guide. These opportunities are entry points for integration, but they should not limit educators from developing reasoning skills where appropriate.

Reasoning Evidence Statements	What does this look like for students in Grades 3 – 5?
R.1 Use visual, symbolic, or contextual representations to explain mathematical ideas.	Students use representations to make their thinking visible and to build a more complete understanding of mathematical ideas. They connect multiple representations to show how the same relationship or idea can be expressed in different ways, and they explain how one representation supports or clarifies another. For example, a student may use an area model and an equation together to explain why two multiplication strategies result in the same product or connect a fraction strip and a number line to show how fractions with different denominators represent the same value.
R.2 Identify and critique errors in reasoning, misapplied procedures, or invalid assumptions, and provide a mathematically valid alternative.	Students use mathematical discussion to evaluate whether reasoning is sound and conclusions are justified. By listening to peers' explanations and asking clarifying or challenging questions, students learn to recognize when a claim depends on an unstated assumption or only works in certain cases. Through these exchanges, students revise their explanations to better reflect the mathematical idea. For example, during a class discussion comparing fractions, a student may claim that $\frac{3}{8}$ is greater than $\frac{1}{2}$ because 3 is greater than 1. Another student questions this reasoning by pointing out that the pieces are different sizes and asks how the fractions compare when the same whole is considered. As the discussion continues, the class revises the explanation to reflect a more accurate comparison.

Reasoning Evidence Statements	What does this look like for students in Grades 3 – 5?
R.3 Construct coherent chains of reasoning that connect mathematical properties, ideas, and evidence, such as examples or counterexamples, to support or refute mathematical statements or claims.	Students support mathematical claims by explicitly linking ideas so that their reasoning is clear and convincing. They connect definitions, properties, observations, or examples to show how multiple ideas work together to justify a conclusion. Rather than presenting isolated statements or answers, students explain how their ideas are connected to support or refute a claim. For example, when explaining why two fractions are equivalent, a student may connect the idea that both fractions represent the same amount of a whole with the observation that the whole is partitioned differently but the shaded portions cover the same area. They use both ideas together to justify why the fractions are equal.
R.4 Identify and extend patterns or regularities to form general rules and justify their validity using mathematical reasoning.	Students use repeated reasoning to move from noticing patterns to forming general rules. The focus is not on predicting the next term in a sequence, but on explaining <i>why the same relationship holds across many cases</i>. Students identify what remains consistent and connect that regularity to underlying mathematical ideas. This supports them in justifying a rule that applies beyond individual examples. For example, a student may notice that multiplying a whole number by 10 always changes the value in the same way. The student explains that each digit represents ten times as much as the digit to its right, so multiplying by 10 shifts every digit one place to the left, which increases the value of the number by a factor of ten. By applying this reasoning to several numbers, the student justifies that the rule holds for all whole numbers.

Numeracy Development Framework

Grade 3 marks a pivotal point in students' numeracy development as they move from primarily additive ways of thinking toward reasoning with multiplicative structure, units, and relationships. Across domains, students increasingly see quantities as composed of equal groups, parts of wholes, and measurable units, and they use representations and reasoning to make sense of mathematical situations. This year establishes essential foundations for proportional reasoning and algebraic thinking in later grades.

Students' work with whole numbers expands in service of this shift. Place value understanding supports flexible reasoning with larger quantities and estimation, while multiplication and division become central tools for describing how quantities grow, partition into equal groups, and scale in relation to one another. Fluency develops alongside meaning as students make use of structure and properties to reason efficiently and assess the reasonableness of results.

Fractions introduce a new lens on quantity that deepens students' understanding of units and relationships. Students interpret fractions as equal parts of a whole and reason about their size using visual models and benchmarks. A key insight at this grade is that the size of a fractional part depends on the whole being considered—an idea that supports later reasoning about equivalence, comparison, and proportion.

In Algebraic Thinking, students extend their use of symbols and equations to represent multiplicative relationships. They model situations using equations with unknowns, solve for unknown quantities, and attend to numerical patterns that reflect predictable relationships between quantities. This work strengthens students' ability to generalize relationships and reason relationally.

Geometric Reasoning and Measurement reinforce multiplicative and unit-based thinking. Students attend to structure when classifying shapes and determining measures such as perimeter and area as they connect geometric reasoning to multiplication. Measurement contexts provide opportunities to reason about units, iteration, and scale in applied ways.

In Reasoning with Data, Statistics, and Probability, students organize and represent data to make sense of contexts and identify patterns. They use these patterns to make and evaluate generalizations, and strengthen habits of reasoning with evidence and attending to whether conclusions are reasonable.

The developmental milestones identified in the Numeracy Development Framework for Grade 3 reflect a coherent move toward multiplicative reasoning, fractional understanding, and relational thinking across domains. This companion document highlights how these ideas align with Grade 3 standards and points to the understandings most critical to monitor as students build toward proportional reasoning, algebra, and measurement in the upper elementary grades.

Number and Operation Sense (NOS)

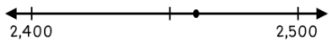
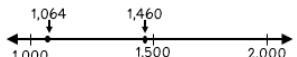
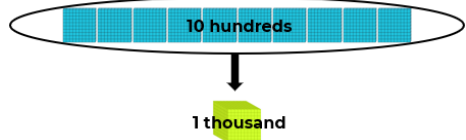
Number and Operation Sense (NOS) Clusters	Connections to the Numeracy Development Framework
3.NOS.A Understand place value.	Standards within this cluster directly contribute* to the Grade 3 Number and Operation Sense milestones within the Numbers & Number Sets strand of the Numeracy Development Framework.
3.NOS.B Represent and interpret problems involving multiplication and division.	Standards within this cluster contribute** to the Grade 3 Number and Operation Sense milestones within the Operations strand of the Numeracy Development Framework.
3.NOS.C Understand properties of multiplication and the relationship between multiplication and division.	Standards within this cluster directly contribute* to the Grade 3 Number and Operation Sense milestones within the Operations strand of the Numeracy Development Framework.
3.NOS.D Use place value understanding and properties of operations to perform multi-digit arithmetic.	Standards within this cluster directly contribute* to the Grade 3 Number and Operation Sense milestones within the Estimation strand of the Numeracy Development Framework.
3.NOS.E Multiply and divide within 100.	Standards within this cluster directly contribute* to the Grade 3 Number and Operation Sense milestones within the Operations strand of the Numeracy Development Framework.
3.NOS.F Develop understanding of fractions as numbers.	Standards within this cluster directly contribute* to the Grade 3 Number and Operation Sense milestones within the Fractions & Decimals strand of the Numeracy Development Framework.

***Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

****Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.

3.NOS.A UNDERSTAND PLACE VALUE.**3.NOS.A.1: Apply and extend place value understanding to whole numbers within 10,000**

- Read and write whole numbers using base-ten numerals, number names, and expanded form.
- Compose and decompose whole numbers.
- Estimate quantities by reasoning about their location on a number line, their relationship to benchmark numbers, and rounding to nearest 10 or 100.
- Compare two whole numbers by reasoning about the values of the digits and the location of the numbers on a number line. Record the results of comparisons with the symbols $>$, $=$, and $<$.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Within 1,000: read, write, and compare numbers; compose and decompose numbers; estimate quantities (2.NOS.A.3). Where are students going? - Apply and extend place value understanding to whole numbers within 1,000,000 (4.NOS.A.1).	3.NOS.A.1a: Use base-ten numerals (1,536), number names (one thousand, five hundred thirty-six), and expanded form ($1,000 + 500 + 30 + 6$) flexibly. 3.NOS.A.1b: Compose and decompose numbers in a variety of ways. For example, 3,672 can be decomposed by place value as $3,000 + 600 + 70 + 2$ or using compatible numbers such as $3,500 + 172$. 3.NOS.A.1c: Estimation is flexible. The number line shown below helps a student to visualize 2,461 as a number between 2,400 and 2,500 but that is also close to 2,450. Using a benchmark number like 2,450 may be more useful in certain contexts, while rounding to 2,500 may be useful in others.  3.NOS.A.1c: When rounding, consider the value of the number and its relationship to benchmarks. For example, 2,461 rounded to the nearest 100 is 2,500 because 2,461 is closer to 2,500 as shown above. Avoid using tricks (e.g., “five and above, give it a shove”) without connecting to number relationships. 3.NOS.A.1d: Use number lines to visualize the relationship between numbers to compare. The number line shows that $1,460 > 1,064$ because 1,460 is further to the right than 1,064. 	- Extend their understanding of the place value to explore and recognize patterns, including 10 hundreds is equal to 1,000 and 10 thousands is equal to 10,000 as they begin to work with greater numbers (SMP 7).  - Use a variety of representations to read, write, and visualize numbers, making connections between the representations. - Explore the many ways to compose and decompose numbers and discuss why certain ways may be more useful in particular situations. - Think flexibly about numbers and quantities. Estimation should be frequently embedded into discussions about numbers and their relationships. - Use horizontal and vertical number lines to justify and explain how to round a given number to the nearest 10 or 100 or to compare a set of numbers. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Number and Operation Sense milestones in the Numbers & Number Sets strand of the Numeracy Development Framework.

Adopted 2025

3.NOS.B REPRESENT AND INTERPRET PROBLEMS INVOLVING MULTIPLICATION AND DIVISION.

3.NOS.B.2: Represent and interpret multiplication of two factors (0-10) as equal groups or as a composed unit that can be repeated/iterated. Explain the relationship between the factors and products.

Content Progression	Examples and Clarifications	Students should have the opportunity to...						
Where are students coming from? - Use skip-counting to find the total number of objects arranged in rectangular arrays(2.NOS.A.2b). Where are students going? - Multiply whole numbers, including 4-digit by 1-digit and 2-digit by 2-digit (4.NOS.B.5) - Explain and apply concepts of factors, multiples, prime, and composite numbers (4.NOS.C.8). - Interpret a multiplication situation as a comparison (4.AT.A.1).	- As students transition from additive reasoning to multiplicative reasoning, they begin solving problems that involve equal groups. This requires students to recognize a group of items as a unit that can be repeated or iterated. - In equal group contexts, students need to consider the number of groups, number in each group, and the total. - When solving the problem, “how many fingers are on 3 hands?”, a student recognizes a hand as one group of 5 fingers. The unit can be repeated or iterated to find the total number of fingers. One factor represents the number of groups (3 hands) and the other factor represents the number in each group (5 fingers).  - The focus of this standard is for students to interpret multiplication expressions/equations or representations using the meaning of multiplication. Avoid solely asking students to identify the product. - Students are not expected to use the term “iterate.”	- Use everyday objects (e.g., headphones to show groups of 2, chairs to show groups of 4) to make sense of and represent equal groups and practice repeating or iterating units. - Make sense of quantities and their relationship to one another. Students should be encouraged to connect expressions and equations to the meaning of multiplication. For example, a student may explain that “3 × 2 means that there are 3 groups of 2.” (SMP 2). - Transition from counting by ones and using repeated addition (additive reasoning) to skip counting or counting multiples to find the total or product (multiplicative reasoning). <table border="1"><thead><tr><th>Additive Reasoning</th><th>Multiplicative Reasoning</th></tr></thead><tbody><tr><td> Counting by ones </td><td> Skip counting </td></tr><tr><td> Repeated Addition  5 + 5 + 5 = 15 </td><td> Multiplication  3 × 5 = 15 </td></tr></tbody></table> - Discuss the relationship between factors and products. Encourage students to make connections to what the factors and product mean in relation to a given context (SMP 2).	Additive Reasoning	Multiplicative Reasoning	Counting by ones 	Skip counting 	Repeated Addition  5 + 5 + 5 = 15	Multiplication  3 × 5 = 15
Additive Reasoning	Multiplicative Reasoning							
Counting by ones 	Skip counting 							
Repeated Addition  5 + 5 + 5 = 15	Multiplication  3 × 5 = 15							

Previously 3.OA.A.1

3.NOS.B.3: Represent and interpret whole-number quotients (0-10) as dividing an amount into known number of groups (partitive) or as dividing an amount into groups of known size (quotative). Explain the relationship between the quotient, divisor, and dividend.

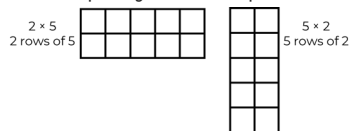
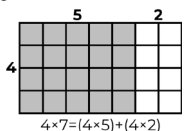
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Use skip-counting to find the total number of objects arranged in rectangular arrays (2.NOS.A.2b). Where are students going? - Divide whole numbers with up to 4-digit dividends and 1-digit divisors without remainders (4.NOS.B.6).	- Partitive division refers to situations where the total and the number of groups are known. For example, there are a total of 12 crayons to be shared equally by 2 students. How many crayons will each student receive? - Quotative division (also called measurement division) refers to situations where the total and the size of the groups are known. For example, there are a total of 12 crayons. Each student will receive 2 crayons. How many students will receive crayons? - While students do not need to use the terms, partitive and quotative, it is important that students have experience with both types and can recognize the difference in these problem types. Strategies will vary based on the type of problem. - Present situations that involve zero (e.g., How can you share 10 cookies with 0 children?). Make connections to the inverse relationship between multiplication and division (e.g., What multiplied by 0 will equal 10?). These discussions will support students in understanding why division by 0 is undefined. Avoid making statements that can lead to misconceptions. Continued on next page	- Use everyday objects to represent partitive and quotative division situations. - Analyze a variety of contextual problems to recognize similarities and differences. - Make sense of quantities and their relationship to one another. Students should be encouraged to connect expressions and equations to the meaning of division. For example, a student may explain that "$12 \div 2$ means that 12 is being divided into 2 group or that 12 is being divided into groups of 2" (SMP 2). - Discuss the relationship between the quotient, divisor, and dividend. Encourage students to make connections to what the quotient, divisor, and dividend mean in relation to a given context (SMP 2).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	The focus of this standard is for students to interpret division expressions/equations or representations using the meaning of division. The focus is not on calculating the quotient.	

Previously 3.OA.A.2

3.NOS.C UNDERSTAND PROPERTIES OF MULTIPLICATION AND THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION.

3.NOS.C.4: Identify and apply the Commutative Property of Multiplication, Associative Property of Multiplication, and Distributive Property of Multiplication as strategies to multiply.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Skip count to find the total number of objects arranged in rectangular arrays (2.NOS.A.2b). Where are students going? - Recall or quickly derive multiplication and division facts within 100 (3.NOS.E.9; 4.NOS.C.7). - Multiply and divide whole numbers (4.NOS.B.5; 4.NOS.B.6).	- Students are expected to describe, explain, and apply the properties. Students are not required to name the properties. Commutative Property of Multiplication: Changing the order of the factors does not change the product (e.g., if $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known). Associative Property of Multiplication: When multiplying three or more factors, the factors can be grouped in any way without changing the product (e.g., $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$). Distributive Property of Multiplication: Multiplying a number by a sum is the same as multiplying the number by each addend separately and adding the products (e.g., knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2)$ which leads to $40 + 16 = 56$). - Intentionally select numbers so calculations do not exceed Grade 3 content limits. Students are expected to work with factors between 0-10 (3.NOS.B.2) and/or with one factor that is a multiple of 10 in the range of 10-90 (3.NOS.D.8). - Provide examples that support students in recognizing the usefulness of the Associative Property of Multiplication. For example, when solving $8 \times 2 \times 4$, solving $(8 \times 2) \times 4 = 16 \times 4$ may be challenging for Grade 3 students. However, $8 \times (2 \times 4) = 8 \times 8$ would be simpler for students to solve. - Model the use of grouping symbols, such as parentheses, to accurately record calculations. However, students are not required to use grouping symbols when writing equations.	- Explore arrays to discover, apply and explain the Commutative Property of Multiplication (SMP 7).  - Explore problems in context to discover, apply and explain the Associative Property of Multiplication (SMP 7). For example, students can determine the cost of water bottles using different groupings of factors: Water bottles are \$2 each. There are 4 bottles in each pack. What is the cost of 5 packs? Students may find the cost of a pack and then multiply by 5 to find the total cost, $(4 \times 2) \times 5 = 40$. Students may also find the total number of water bottles and then multiply by cost to find the total cost, $(4 \times 5) \times 2 = 40$. - Explore area models to discover and demonstrate the Distributive Property of Multiplication. Connect visual representations of the Distributive Property of Multiplication to symbolic notation.  - Construct coherent chains of reasoning that use mathematical properties to support or refute mathematical statements. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Number and Operation Sense milestones in the Operations strand of the Numeracy Development Framework.

Previously 3.OA.B.5

3.NOS.C.5: Explain and use the inverse relationship between multiplication and division to determine the unknown whole number in a multiplication or division equation involving three whole numbers (e.g., find $32 \div 8$ by thinking of unknown-factor problem and finding the number that equals 32 when multiplied by 8).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Skip count to find the total number of objects arranged in rectangular arrays (2.NOS.A.2b). Where are students going? - Multiply and divide whole numbers (4.NOS.B.5; 4.NOS.B.6).	- Students are familiar with the inverse relationship between addition and subtraction. Students will explore and discover how the number of groups, number in each group, and the total are related in multiplication and division contexts. - Students are expected to describe, explain, and apply the inverse relationship between multiplication and division. Students are expected to use terms, inverse relationship or inverse operations. - Use concrete models and visual representation to develop this understanding. Avoid using fact families as the only means for introducing the inverse relationship between multiplication and division as this can cause misconceptions. The diagram shows three 3x5 rectangular arrays. The first array has a '3' to its left and a '5' above it, with a circle containing '15' in the center. The second array has a '?' to its left and a '5' above it, with a circle containing '15' in the center. The third array has a '3' to its left and a '?' above it, with a circle containing '15' in the center. - Students should work with factors and quotients 0-10, in alignment with 3.NOS.B.2 and 3.NOS.B.3. - Use equations with the equal sign in varying positions (e.g., $15 = 3 \times 5$, $3 \times 5 = 15$).	- Explore contextual problems to make sense of the inverse relationship between multiplication and division. Interpret the meaning of numbers and operations (SMP 2). - Use multiple representations to demonstrate the inverse relationship between multiplication and division. - Apply the inverse relationship between multiplication and division to solve for an unknown in an equation (e.g., $? \times 8 = 32$ or $32 \div ? = 8$). - Explain what it means for multiplication and division to be inverse operations.

Previously 3.OA.A.4; 3.OA.B.6

3.NOS.D USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC.

3.NOS.D.6: Estimate sums and differences within 1,000 using benchmark numbers and front-end estimation. Determine when an estimate is appropriate and when an exact answer is needed.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Estimate quantities to assess reasonableness of sums and differences (2.NOS.A.5). Where are students going? - Estimate sums, differences, products, and quotients of multi-digit numbers (4.NOS.B.3a; 4.NOS.B.5a; 4.NOS.B.6a).	- Benchmark numbers are familiar reference points. Often, benchmark numbers are multiples of 10, but they can also be other numbers that are familiar, such as 25 or 75. For example, a student estimating the difference of $618 - 291$ may round to the nearest hundred or may choose other benchmark numbers. $618 - 291 \rightarrow 600 - 300 = 300$ $618 - 291 \rightarrow 625 - 300 = 325$ Both are reasonable estimates but 325 is closer to actual difference of 327. - Front-end estimation is a form of estimation that focuses on using the leading digit of the numbers. For example, if using front-end estimation to estimate the difference of $754 - 423$, a student would rename the expression as $700 - 400$. - Estimation should be flexible. Avoid modeling or requiring prescriptive processes for estimation. - Pose “is it reasonable?” questions regularly to build estimation habits and to encourage students to consider reasonableness. - Provide a variety of contexts and discuss when an estimate would be appropriate and when an exact answer is needed. For example, a Physical Education teacher can estimate how many jump ropes need to be purchased for the upcoming unit, however, an exact answer is needed to identify the total cost when paying for the jump ropes.	- Make decisions about what strategy to use when estimating sums and differences (SMP 5). - Explain strategy for estimation. Students should be encouraged to use visual representations - Discuss the reasonableness of estimations. For example, students may discuss which of the possible estimates is most reasonable given a quantity. Students may consider a specific context when assessing reasonableness. - Estimate before solving addition or subtraction problems to assess the reasonableness of an answer. For example, when solving $239 + 244$, a student may identify that both numbers are close to 240, so the sum should be close to 480 because $240 + 240 = 480$. - Apply understanding of rounding to the nearest 10 and 100 to estimate sums and differences (3.NOS.A.1c). - Apply estimation in authentic contexts (e.g., measurement, time, money). Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Number and Operation Sense milestones in the Estimation strand of the Numeracy Development Framework.

Adopted 2025

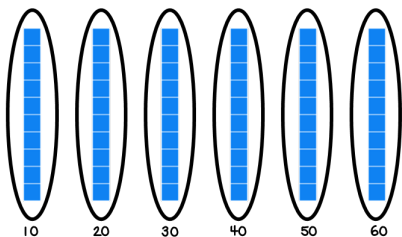
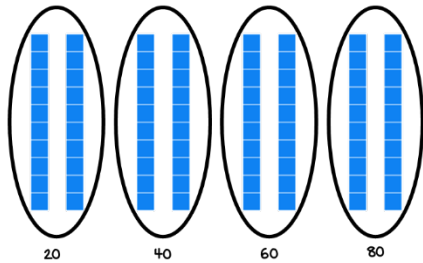
3.NOS.D.7: Fluently add and subtract within 1,000.

- a. Use computational strategies efficiently (e.g., decomposition, partials sums, compensation, make ten/hundred/thousand) to add and subtract.
- b. Use properties of operations, and/or the inverse relationship between addition and subtraction.
- c. Determine and explain when a strategy is most efficient.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Fluently add and subtract within 100 (2.NOS.C.8). - Add and subtract within 1000 using models, drawings, strategies, properties, and inverse relationship (2.NOS.C.9). Where are students going? - Fluently add and subtract multi-digit whole numbers within 1,000,000 (4.NOS.B.3).	- Model the various approaches and encourage flexibility. Avoid modeling or requiring a single approach. - Intentionally select numbers that encourage the use of specific strategies. This supports conceptual understanding and promotes efficient approaches rather than rote calculation. Avoid numbers that distract from mathematical focus by making computation unnecessarily cumbersome. 3.NOS.D.7a: Partial sums involves decomposing addends by place value then adding hundreds to hundreds, tens to tens, ones to ones, and adding the partial sums to find the total. For example, a student may show the steps shown below to solve $237 + 548$. $237 = 200 + 30 + 7 \text{ and } 548 = 500 + 40 + 8$ $7 + 8 = 15$ $30 + 40 = 70$ $200 + 500 = 700$ $700 + 70 + 15 = 785$ 3.NOS.D.7a: Compensation involves changing or adjusting numbers to make a “friendlier” problem. For example, when solving the problem $263 - 127$, a student may add 3 to both numbers to rewrite the problem as $266 - 130$. By adding the same amount to each number, the difference remains the same. 3.NOS.D.7c: When determining efficiency, consider the quantities, their relationships to benchmark numbers, and how those relationships influence strategy choice. Use think-alouds to model this decision-making process.	- Make explicit connections between different approaches. - Use strategies flexibly, making decisions on which strategy or method to use based on the numbers or context of the given problem (SMP 5). - Explain their reason for selecting a particular strategy. For example, a student may explain “when solving $329 + 436$, I used the use ten strategy because 329 is close to 330. I took 1 from 436 and added it to 329 to make 330. Then, I added $330 + 435 = 765$.” - Engage in Number Talks to promote flexible thinking and provide opportunities to explain their thinking and make connections to other strategies. - Apply estimation strategies to assess the reasonableness of sums and differences (3.NOS.D.6).

Previously 3.NBT.A.2

3.NOS.D.8: Multiply one-digit whole numbers by multiples of 10 in the range of 10-90 (e.g., 9×80 ; 60×5) using strategies based on place value and properties of operations.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Compose and decompose three-digit numbers by place value (2.NOS.A.1) Where are students going? - Multiply and divide whole numbers (4.NOS.B.5; 4.NOS.B.6).	- Students will apply their understanding of place value, the meaning of multiplication, and the Associative Property of Multiplication to multiply one-digit whole numbers by multiples of ten. - The expression, 6×10, can be thought of as 6 groups of 10 and can be represented with a place value model. Make connections to skip counting to find the total.  - Similar models will be used when introducing multiplication by multiples of ten. Students should continue to make connections to place value and equal groups. For example, 4×20 can be thought of as 4 groups of 2 tens because 2 tens are equal to 20.  - Avoid teaching students that when multiply by ten (or a multiple of ten), a zero is added to the end of the number. This “rule” can promote misconceptions, especially as students begin working with decimals. For example, that “rule” does not apply when multiplying 1.2×10.	- Use place value models to represent multiplication of one-digit whole numbers by multiples of ten. - Recognize and explain patterns noticed when multiplying a one-digit whole number by multiples of ten (SMP 7). Use and connect visual and symbolic representations to explain these patterns. - Apply understanding of the Associative Property of Multiplication to multiply one-digit whole numbers by multiples of 10. For example, a student may use the equation below to represent and solve 30×3. $30 \times 3 = (3 \times 10) \times 3 = (3 \times 3) \times 10 = 9 \times 10 = 90$ - Engage in Number Talks to promote flexible thinking and provide opportunities to explain their thinking and make connections to other strategies.

Previously 3.NBT.A.3

3.NOS.E MULTIPLY AND DIVIDE WITHIN 100.

3.NOS.E.9: Recall or quickly derive multiplication and division facts within 100 (e.g., factors less than or equal to 10 and quotients less than or equal to 10).

- Skip count (2s, 5s, 10s) and apply properties of operations (0s, 1s) to derive foundational facts.**
- Derive unknown facts from known facts using double facts (e.g., to solve 4×3 , double 3 to get 6 and double 6 to get 12) to multiply and divide.**
- Use properties of operations (e.g., to solve 8×7 , think of $(5 \times 7) + (3 \times 7)$) to multiply.**
- Use the inverse relationship between multiplication and division (e.g., think of $4 \times ? = 28$ to solve $28 \div 4 = ?$) to multiply and divide.**

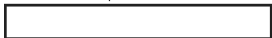
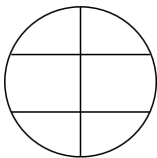
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Skip count to find the total number of objects arranged in rectangular arrays (2.NOS.A.2b). Where are students going? - Recall or quickly derive multiplication and division facts within 100 (4.NOS.C.7). Note: 3.NOS.E.9 focuses on developing mastery of multiplication and division facts within 100. This work is foundational and will continue into Grade 4, where mastery expectations are solidified (4.NOS.C.7).	- Students begin using skip counting as a strategy to solve multiplication and division facts (e.g., to solve 4×2, a student may skip count 2, 4, 6, 8). - Students will also apply the Identity Property of Multiplication (the product of any number and 1 is that number) and the Zero Property of Multiplication (any number multiplied by 0 is 0) to derive facts. Students are expected to describe, explain, and apply the properties but are not required to name them. - Model and encourage students to use more efficient strategies as they work toward mastery. For example, if a student does not know the product of 4×7, they can use known facts to determine the unknown fact. The factor, 7, can be decomposed into $5 + 2$. Then, 4 can be multiplied by each addend separately and the two products can be added: $4 \times 7 = (4 \times 2) + (4 \times 5)$. - Students are expected to recall multiplication and division facts within 100 (where the factors or quotient is 0-10) or to use an efficient strategy to quickly find the product or quotient of an unknown fact. - Use equations with equal sign in varying positions (e.g., $6 = 2 \times 3$, $20 \div 5 = 4$, etc.). - Provide a variety of opportunities for students to practice and demonstrate mastery. Avoid relying solely on timed tests and memorization.	- Engage in explicit strategy instruction, that allows time for exploration and practice. - Use strategies flexibly, making decisions on which strategy or method to use based on the numbers in the problem and what facts the student knows. - Engage in Number Talks to promote flexible thinking and provide opportunities to explain their thinking and make connections to other strategies. - Construct coherent chains of reasoning to explain their strategy for finding a given product or quotient or proving their product or quotient is correct. Engaging in discussion around various strategies allows students still using counting strategies to make connections to more sophisticated strategies (SMP 2, SMP 3). - Discuss patterns and structure they notice while multiplying or dividing, allowing students to make generalizations about how strategies can help in learning multiplication and division facts (SMP 7). Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Number and Operation Sense milestones in the Operations strand of the Numeracy Development Framework.

Previously 3.OA.C.7

3.NOS.F DEVELOP UNDERSTANDING OF FRACTIONS AS NUMBERS.

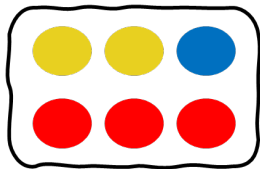
(Limit to fractions with denominators 1, 2, 3, 4, 6, 8)

3.NOS.F.10: Recognize and explain that a fraction represents a part of a whole.**a. Partition shapes into parts with equal areas and express the area of each part using a unit fraction.****b. Explain that a fraction $\frac{1}{d}$ represents one part of a whole that is divided into d equal parts, and that a fraction $\frac{c}{d}$ represents c parts of size $\frac{1}{d}$.**

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Partition circles and rectangles into equal shares (2.NOS.D.11). Where are students going? - Add and subtract fractions (4.NOS.E.11, 4.NOS.E.12). - Multiply a fraction by a whole number (4.NOS.E.13).	- Grade 3 students will work with denominators of 1, 2, 3, 4, 6, and 8. - A fraction is a number that represents a part of a whole. Fractions are written with a numerator (tells the number of parts) and a denominator (tells how many equal parts the whole is partitioned into). $\frac{1}{2}$ 1 ← numerator 2 ← denominator - A unit fraction represents one equal part of a whole that has been divided into equal parts. A unit fraction will always have a numerator of 1. - Build students' conceptual understanding by connecting fractions to contexts that are familiar to students, such as sharing a candy bar. Students may represent fractions using physical objects, manipulatives, drawings, and fraction notation. - Provide examples of both pre-partitioned and unpartitioned representations. pre-partitioned  unpartitioned  - Provide examples of fractions less than 1, equal to 1 and greater than 1. Use "fractions greater than 1." Avoid using "improper fractions" as this phrase can be confusing/misleading.	- Partition shapes in multiple ways to represent halves, thirds, fourths, sixths, and eighths and use fraction notation to name each equal part. - Explain the meaning of the numerator and denominator. - Make connections between multiple visual representations of fraction and fraction notation. - Use examples and counterexamples to justify or refute mathematical claims. For example, a student may use the counterexample shown below to refute the statement that all shapes cut into six pieces represent sixths (SMP 3).  - Make connections to time when discussing halves and quarters (3.GR.A.2). Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Number and Operation Sense milestones in the Numbers & Number Sets strand of the Numeracy Development Framework.

Previously 3.NF.A.1; 3.G.A.1

3.NOS.F.11: Explain a fraction of a set $\frac{c}{d}$ as the quantity formed by c items as part of the given set of d objects.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Partition circles and rectangles into equal shares (2.NOS.D.11). Where are students going? - Understand that comparisons are only valid when the two fractions refer to the same whole (4.NOS.D.10). - Apply knowledge of whole number operations to fraction operations (4.NOS.E.11, 4.NOS.E.12, 4.NOS.E.13).	- Grade 3 students will work with denominators of 1, 2, 3, 4, 6, and 8. - A set model is a set of objects that represents the whole. For example, in the set below, the whole is the 6 counters.  - Students are expected to use fractions to describe a given set. In the example above, a student may describe the set as one-sixth of the counters are blue, two-sixths of the counters are yellow, and three-sixths of the counters are red. Some students may recognize that you could also describe the red counters as one-half of the set, since three-sixths and one-half are equivalent. Students are not expected to calculate the exact number of objects that correspond to a fractional part of the set. (e.g., students are not expected to determine that one-half of the given set is 3 counters).	- Explore set models through hands-on experiences with everyday objects and manipulatives. Students may create sets and identify fractional parts of the sets. - Explore multiple examples of sets with varying wholes. - Represent fractions of a set using visual models such as drawings or arrays and using fraction notation. - Use fractions to describe everyday situations that involve sets (e.g., two-eighths of the apples are green).  - Explain the part-whole relationship of a given set model. In the example above, a student may explain that the whole is the set of eight apples and two-eighths of the apples are green (SMP 2).

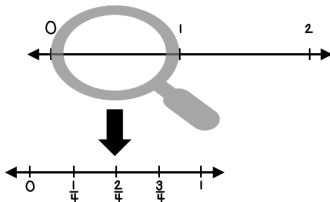
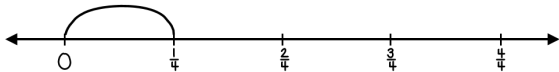
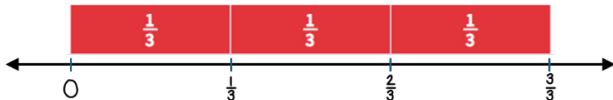
Adopted 2025

3.NOS.F.12: Represent a fraction as a number on the number line.

- a. Represent a fraction $\frac{1}{d}$ on a number line by defining the interval from 0 to 1 as the whole and partitioning it into d equal lengths.

Recognize that each length measures $\frac{1}{d}$ and that the endpoints of the length starting at 0 represents the number $\frac{1}{d}$ on the number line.

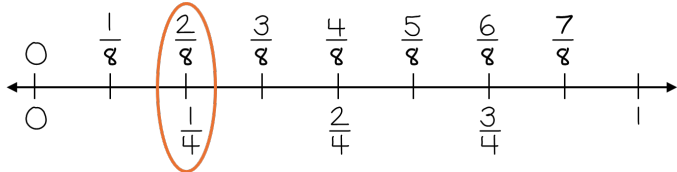
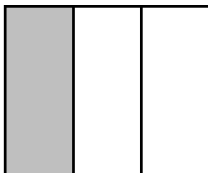
- b. Represent a fraction $\frac{c}{d}$ on a number line by marking off c lengths of $\frac{1}{d}$ from 0. Recognize that the resulting interval has size $\frac{c}{d}$ and that the endpoint of the length locates the number $\frac{c}{d}$ on the number line.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Use number lines to compare numbers, estimate quantities, and represent sums and differences (2.NOS.A.4, 2.NOS.A.5, 2.NOS.C.7). Where are students going? - Use visual models to solve addition, subtraction, and multiplication problems in context (4.NOS.E.12, 4.NOS.E.13). - Locate decimals on a number line (4.NOS.F.15).	- Grade 3 students will work with denominators of 1, 2, 3, 4, 6, and 8. - A number line shows units of length rather than discrete points. - Number lines are used to represent fractions, including fractions greater than 1. Support students in recognizing that the space between whole numbers can be further partitioned and those lengths can be represented by fractions.  - Emphasize the space from 0 to a given number and not just the tick mark for the given number. For example, emphasize the space from 0 to $\frac{1}{4}$ when representing $\frac{1}{4}$ on a number line.  - Present horizontal and vertical number lines to support students' flexible use of this tool. - Use "fractions greater than 1." Avoid using "improper fractions" as this phrase can be confusing/misleading.	- Engage with physical number lines. For example, students may move along a large number line taped to the classroom floor or move an object along a large number line on classroom wall. - Make connections between fraction bars, area models, and number lines.  - Use fraction bars to create their own number lines to emphasize equal lengths (SMP 6). Model and discuss precision when creating number lines. - Use number lines to interpret and describe quantities, recognizing that fractions can represent distances from zero and from each other (SMP 2). - Skip count by unit fractions using a number line as a visual support. Encourage students to identify and discuss patterns. - Extend the number line beyond 1 and discuss patterns. - Apply fractions represented on a number lines to authentic contexts (e.g., measurement). Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Number and Operation Sense milestones in the Fractions & Decimals strand of the Numeracy Development Framework.

Previously 3.NF.A.2

3.NOS.F.13: Explain equivalence of fractions and compare fractions by reasoning about their size, recognizing that comparisons are valid only when the two fractions refer to the same whole.

- Represent two fractions as equivalent if they are the same size, or they represent the same point on the number line.
- Identify and generate equivalent fractions, (e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$) and explain why the fractions are equivalent using concrete materials, drawings, number lines, or equations.
- Express whole numbers as fractions, and identify fractions that are equivalent to whole numbers, (e.g., express 3 in the form $3 = \frac{3}{1}$; recognize that $6 = \frac{6}{1}$; locate $\frac{4}{4}$ and 1 at the same point on a number line).
- Compare two fractions with the same numerator or the same denominator by reasoning about their size. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions (e.g., by using concrete materials, drawings, number lines, and/or equations).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Partition circles and rectangles into equal shares (2.NOS.D.11). Where are students going? - Understand and generate equivalent fractions (4.NOS.D.9) - Compare two fraction with different numerators and denominators (4.NOS.D.10).	- Grade 3 students will work with denominators of 1, 2, 3, 4, 6, and 8. - Equivalent fractions represent the same quantity or same length on a number line. - Students should use equivalent fractions strategically and flexibly. Students are not required to simplify fractions, however, they should be able to recognize equivalent fractions when necessary. 3.NOS.F.13b: Concrete and visual models are used to identify, generate, and explain equivalent fractions. Equations may be used to represent the relationship between equivalent fractions. Grade 3 students are not expected to use procedures or algorithms to generate equivalent fractions. For example, a student may use the number line below to explain that $\frac{2}{8} = \frac{1}{4}$ because they represent the same length on the number line.  <i>Continued on next page</i>	- Explore a variety of concrete and visual models to develop conceptual understanding of fraction equivalence. - Use visual representations to explain why fractions are equivalent, making connections between models and equations to explain their reasoning. - Look for and identify patterns in the numerator and denominator of two or more equivalent fractions (e.g., what patterns do you notice between the equivalent fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$?) (SMP 7). - Engage in concrete experiences to express whole numbers as fractions. - Use examples and counterexamples to justify or refute mathematical claims. For example, a student may use the counterexample shown below to refute the statement that one-half is always greater than one-fourth (SMP 3).  

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	3.NOS.F.13c: Present concrete and visual models as examples to support students in recognizing that any whole number can be expressed as a fraction. For example, 4 can be represented as $\frac{4}{1}$ because it is 4 wholes each partitioned into 1 equal piece. In the visual model shown below, 4 can also be represented as $\frac{8}{2}$ because it is eight pieces of size one-half. 3.NOS.F.13d: When comparing fractions with the same numerator, students use concrete or visual models to explain that since the number of pieces (numerator) is the same, the size of the pieces (denominator) is used to compare. 3.NOS.F.13d: When comparing fractions with the same denominator, students use concrete or visual models to explain that since the size of pieces (denominator) is the same, the number of the pieces (numerator) is used to compare. - Students are not expected to identify equivalent fractions of a set or compare fractions of a set. - Use “fractions greater than 1.” Avoid using “improper fractions” as this phrase can be confusing/misleading.	Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Number and Operation Sense milestones in the Fractions & Decimals strand of the Numeracy Development Framework.

Previously 3.NF.A.3

Algebraic Thinking (AT)

Algebraic Thinking (AT) Clusters	Connections to the Numeracy Development Framework
3.AT.A Solve problems involving the four operations and identify and explain patterns in arithmetic.	Standards within this cluster directly contribute* to the Grade 3 Algebraic Thinking milestones within the Creating Equations & Inequalities, Solving Equations & Inequalities, and Functional Reasoning strands of the Numeracy Development Framework.

***Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

****Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.

3.AT.A SOLVE PROBLEMS INVOLVING FOUR OPERATIONS AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC.

3.AT.A.1: Use multiplication and division within 100 to solve one-step problems in context involving equal groups, arrays, and area by using concrete materials, drawings, and/or equations.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Use addition and subtraction within 100 to solve one-step problems in context (2.AT.A.1) Where are students going? - Multiply or divide to solve multiplicative comparison problems in context (4.AT.A.2).	- Grade 3 students should solve problems involving whole numbers and having whole number solutions. Multiplication and division should be within 100 (in alignment with 3.NOS.B and 3.NOS.E). - Multiplication and division problems should be presented in authentic contexts. Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students' everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved. Refer to Multiplication and Division Situations for examples of multiplication and division situations. Grade 3 students will work with <i>equal groups of objects</i> and <i>arrays of objects</i> situations with unknowns in all positions. - Support students in making sense of multiplication and division situations in context. Model and encourage students to retell, act it out, draw pictures, etc. Avoid using key word identification in problems (e.g., CUBES method) as this does not support students' problem solving and may lead to misconceptions. - A variety of representations and strategies can be used to represent and solve multiplication and division problems (in alignment with 3.NOS.B, 3.NOS.C, 3.NOS.D and 3.NOS.E). Avoid modeling or requiring a specific representation or strategy.	- Continue to build understanding of multiplication and division as operations through the exploration of contextual problems. - Describe what is happening in a multiplication and division situation in their own words (SMP 1). - Discuss the meaning of quantities in the presented contexts and how the quantities are related (SMP 2). - Determine the information needed to answer the mathematical question. - Represent multiplication and division situations mathematically by selecting and applying appropriate representations (e.g., manipulatives, drawings, number lines, equations, etc.). Develop flexibility in their use of various strategies. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Algebraic Thinking milestones in the Creating Equations & Inequalities strand of the Numeracy Development Framework.

Previously 3.OA.A.3

3.AT.A.2: Use the four operations to solve two-step problems in context (within number limits for each operation as stated in 3.NOS.B and 3.NOS.D).

- a. Represent these problems using equations with a letter standing for the unknown quantity.
- b. Assess the reasonableness of answers in terms of context, using mental computation and estimation strategies including, rounding, front-end estimation, and benchmark numbers.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Use addition and/or subtraction within 100 to solve twostep problems in context (2.AT.A.2). Where are students going? - Use the four operations to solve multistep problems in context involving whole numbers (4.AT.A.4).	- Grade 3 students should solve problems involving whole numbers and having whole number solutions. Addition and subtraction should be within 1,000 (in alignment with 3.NOS.D.7). Multiplication and division should be within 100 (in alignment with 3.NOS.B, 3.NOS.D.8 and 3.NOS.E). - Problems should be presented in authentic contexts. Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students' everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved. Refer to Situation Types for Operations in Word Problems for examples of addition, subtraction, multiplication, and division situations. Grade 3 students work with addition and subtraction situations with unknowns in all positions. For multiplication and division situations, Grade 3 students work with <i>equal groups of objects</i> and <i>arrays of objects</i> situations with unknowns in all positions. - Support students in making sense of two-step problems in context. Model and encourage students to retell, act it out, draw pictures, etc. Avoid using key word identification in problems (e.g., CUBES method) as this does not support students' problem solving and may lead to misconceptions. <i>Continued on next page</i>	- Continue to build understanding of operations through the exploration of contextual problems. - Describe what is happening in a given context in their own words (SMP 1). - Discuss the meaning of quantities in the presented contexts and how the quantities are related (SMP 2). - Formulate a question about a particular context that can be answered using mathematics. - Determine the information needed to answer the mathematical question. - Represent multiplication and division situations mathematically by selecting and applying appropriate representations (e.g., manipulatives, drawings, number lines, equations, etc.). Develop flexibility in their use of various strategies. - Use a variety of mental computation and estimation strategies to estimate before solving. - Interpret the results in of their mathematical model in terms of the context to determine what the answer means in the situation. - Revise their mathematical model when they determine their answer is not reasonable. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Algebraic Thinking milestones in the Creating Equations & Inequalities and Solving Equations & Inequalities strands of the Numeracy Development Framework.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	• 3.AT.A.2a: It is important for students to consider the order in which operations are performed. Discuss the appropriate order for completing the computations and what happens if they are completed in a different order. Model the use of grouping symbols, such as parentheses, to accurately record calculations. However, students are not required to use grouping symbols when writing equations. • 3.AT.A.2a: Transition students from using a symbol, such as a question mark or box to represent an unknown, to using letters to represent the unknown. When using equations, students may use one or two equations to represent the situation. Using two equations will require students to represent two unknowns. The art teacher has 225 markers and will purchase 6 more packs of markers. Each pack contains 8 markers. How many markers will the art teacher have? $6 \times 8 + 225 = m$ OR $6 \times 8 = \square \text{ and } \square + 225 = m$ • 3.AT.A.2b: Model and encourage students to use mental computation and/or estimation strategies before solving to identify a reasonable answer. After solving, revisit estimate and context to ensure the answer is reasonable. as this may impact the accuracy and reasonableness of an answer. • A variety of representations and strategies can be used to represent and solve two-step problems (in alignment with 3.NOS.B, 3.NOS.C, 3.NOS.D and 3.NOS.E). Avoid modeling or requiring a specific representation or strategy.	

Previously 3.OA.D.8

3.AT.A.3: Identify arithmetic patterns and explain them using properties of operations (e.g., observe the pattern that 4 times a number is always even and explain why 4 times a number can be decomposed into two equal addends).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Determine whether a group of objects within 20 has an odd or even number of members (2.AT.B.3). Where are students going? - Generate and analyze number patterns (5.AT.B.3).	- Students will identify patterns in calculations as well as in the addition and multiplication tables. - Students apply understanding of properties to identify and explain patterns Commutative Property of Multiplication: Changing the order of the factors does not change the product (e.g., if $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known). Associative Property of Multiplication: When multiplying three or more factors, the factors can be grouped in any way without changing the product (e.g., $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$). Distributive Property of Multiplication: Multiplying a number by a sum is the same as multiplying the number by each addend separately and adding the products (e.g., knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5+2) = (8 \times 5) + (8 \times 2)$ which leads to $40+16 = 56$). - Ask students questions, such as “How do you know?” and “Does this always work?”, to support students reasoning.	- Engage in Number Talks to provide opportunities for students to recognize patterns in calculations. - Describe and explain patterns discovered using examples, counterexamples, and appropriate mathematical language. - Anticipate results based on patterns noticed (SMP 8) and explain their thinking. - Make connections to patterns in authentic contexts. For example, if 1 week has 7 days, then 2 weeks have 14 days, 3 have 21, and so on. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Algebraic Thinking milestones in the Functional Reasoning strand of the Numeracy Development Framework.

Previously 3.OA.D.9

Geometric Reasoning and Measurement (GR)

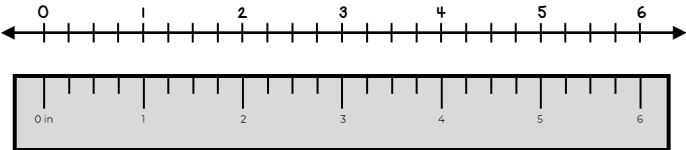
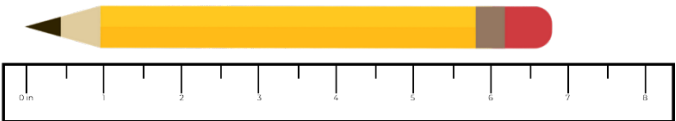
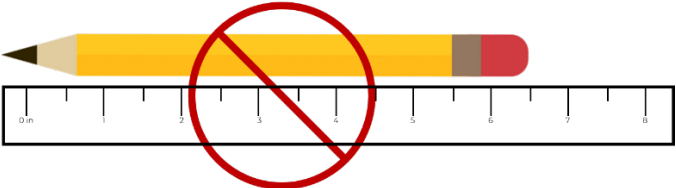
Geometric Reasoning and Measurement (GR) Clusters	Connections to the Numeracy Development Framework
3.GR.A Solve problems involving measurement and estimation of intervals of time, liquid, and masses of objects.	Standards within this cluster directly contribute* to the Grade 3 Geometric Reasoning and Measurement milestones within the Measurement strand of the Numeracy Development Framework.
3.GR.B Reason with shapes and their attributes.	Standards within this cluster directly contribute* to the Grade 3 Geometric Reasoning and Measurement milestones within the Classifying Figures strand of the Numeracy Development Framework.
3.GR.C Understand concepts of area and perimeter.	Standards within this cluster directly contribute* to the Grade 3 Geometric Reasoning and Measurement milestones within the Reasoning with Geometric Structure strand of the Numeracy Development Framework.

***Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

****Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.

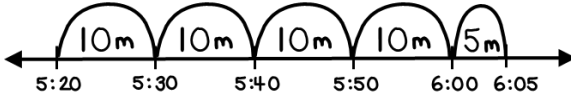
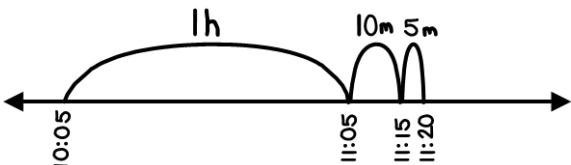
3.GR.A SOLVE PROBLEMS INVOLVING MEASUREMENT AND ESTIMATION OF INTERVALS OF TIME, LIQUID, AND MASSES OF OBJECTS.

3.GR.A.1: Measure the length of an object to the nearest half and quarter unit by using appropriate tools (e.g., rulers, yardsticks, meter sticks, and measuring tapes).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Measure length of object to the nearest whole unit (2.GR.A.1) Where are students going? - Convert measurements from a larger unit to a smaller unit (4.GR.A.1). - Use the four operations to solve problems in context involving measurement (4.GR.A.2).	- Grade 3 students will measure lengths to the nearest half and quarter unit. Select objects whose lengths clearly align with or are close to half- or quarter-unit measurements to support accurate estimation. - Provide rulers that show whole-, half- and quarter units. Support students in making connections between fractions on a number line (3.NOS.F.12) and rulers.  - Record measurements using fractions. For measurements greater than 1, students are not required to use mixed numbers, as they are not formally introduced until Grade 4. However, students may naturally begin using this notation. For example, a student may measure a paper clip and describe its length as one and a half inches. A student may describe the length as $\frac{3}{2}$ inches. Students are not expected to record measurements using decimals. - Students must make decisions about which measuring tool to use when measuring lengths. For example, when measuring the length of the classroom a student may determine that using a yardstick or meter stick would be appropriate but when measuring the length of a desk a ruler would be appropriate. - Present measurement tasks in authentic contexts (e.g., crafts, building, science experiments). Model how to apply estimation strategies to estimate the measurement to nearest half or quarter unit (3.NOS.A.1c).	- Explore using multiple tools when measuring lengths to discover relationships between tools and sizes of objects. - Estimate before measuring lengths to make decisions and select the most appropriate tool (SMP 5). - Practice using measurement tools to ensure precise measurements, aligning the 0 marker on the tool and not just the edge of the measuring tool (SMP 6).   - Use measurement vocabulary when explaining their process and describing attributes of objects. - Compare measurements of the same object using different units (e.g., inches vs. centimeters) and explain similarities/differences. - Apply understanding of measurement to representing fractions on a number line (3.NOS.F.12) and using line plots to display data (3.DS.A.1).

Previously 3.MD.B.4

3.GR.A.2: Solve problems in context that involve addition and subtraction of time intervals in minutes (e.g., represent and solve using a number line).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Tell and represent time to the nearest minute (2.GR.C.7). - Estimate and calculate elapsed time to the hour and half hour (2.GR.C.8). Where are students going? - Use the four operations to solve problems in context involving intervals of time (4.GR.A.2).	- Students continue to develop the concept of time as a measure that tells us when events happen and how long the events last. Clocks (analog and digital), timers and number lines can be used to help students understand time. - This standard emphasizes students' ability to estimate and calculate elapsed time using standard time (12-hour clock format). - Elapsed time is the time that passes between the start of an event and the end of the event. - When calculating elapsed time, students use number lines to represent addition and subtraction of time intervals as in the examples shown below. Avoid overemphasis on rote procedures without connection to visual models. If you begin watching a television show at 5:20 p.m. and the show lasts 45 minutes, you will finish watching at 6:05 p.m.  If math class begins at 10:05 a.m. and ends at 11:20 a.m., the math class lasts 1 hour and 15 minutes.  - Consider the reasonableness of time intervals when selecting or designing problems and tasks. Instruction may extend beyond 120 minutes to allow for exploration, practice, discussion, and connections to authentic contexts. Assessments, however, will not exceed 120-minute intervals.	- Engage in daily routines that support students' understanding of time, including estimating and calculating time intervals. - Use digital clocks, analog clocks, and both partitioned and open number lines when estimating and calculating elapsed time. - Develop their proficiency using number lines by initially making multiple “jumps” to perform operations. As students become more proficient, they will recognize the repeated calculations can be completed more efficiently through “jumps” of larger quantities (SMP 8). - Make sense of time and relationship between times. For example, students explain how a number line can represent how minutes and hours can be added on a number line (SMP 2). - Solve problems in authentic contexts (e.g., school schedules, travel, cooking) and use representations to explain mathematical ideas. Students should have the opportunity to solve a balance of problems with unknown start time, unknown end time, and unknown elapsed time. - Engage in problem contexts that involve calculating earnings based on hours (e.g., babysitting job from 1:15 to 4:45 at \$10 per hour). \$

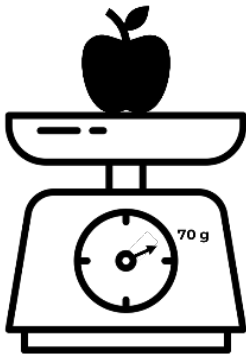
Previously 3.MD.A.1

3.GR.A.3: Measure and estimate liquid volumes and masses of objects using standard units of liters(l), grams (g), and kilograms (kg).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Estimate, measure and compare lengths using standard units (2.GR.A.1, 2.GR.A.2, 2.GR.A.3, 2.GR.A.4). Where are students going? - Convert measurements from a larger unit to a smaller unit (4.GR.A.1). - Use the four operations to solve problems in context involving measurement (4.GR.A.2).	- Liquid volume is the three-dimensional space taken up by a liquid. It can be measured using liters (l). Note, students will identify liquid volume through measurement. Students are not expected to calculate volume using a formula. - Mass is the amount of matter in an object. It can be measured using grams (g) or kilograms (kg). - Grade 3 students will measure to the nearest whole unit. Select objects whose liquid volumes and masses clearly align with or are close to whole unit measurements to support accurate estimation. - Present measurement tasks in authentic contexts (e.g., grocery shopping, cooking, science experiments).	- Engage in hands-on experiences to develop an understanding of liquid volume and mass. For example, a student may fill a 1-liter bottle using a 250-milliliter measuring cup, observing accumulation to develop understanding of liquid volume and support estimation. - Explore using multiple measurement tools, including measuring cups, beakers, graduated cylinders, liter bottles, and balance scales. - Estimate before measuring liquid volumes or masses and select the most appropriate tool (SMP 5). - Use appropriate vocabulary when estimating, measuring, explaining their process, and describing attributes of objects. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Geometric Reasoning & Measurement milestones in the Measurement strand of the Numeracy Development Framework.

Previously 3.MD.A.2

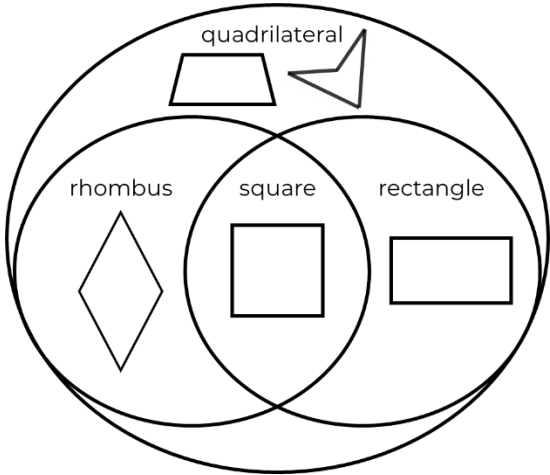
3.GR.A.4: Add, subtract, multiply, or divide to solve one-step problems in measurement contexts involving masses or volumes that are given in the same units.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Estimate, measure and compare lengths using standard units (2.GR.A.1, 2.GR.A.2, 2.GR.A.3, 2.GR.A.4). Where are students going? - Convert measurements from a larger unit to a smaller unit (4.GR.A.1). - Use the four operations to solve problems in context involving measurement (4.GR.A.2).	- Grade 3 students will solve one-step problems involving the wholes numbers of the same unit. Grade 3 students are not expected to convert measurements from one unit to another unit. - Computation should not exceed Grade 3 expectations (3.NOS.B, 3.NOS.D, and 3.NOS.E). - Present measurement tasks in authentic contexts (e.g., grocery shopping, cooking, science experiments). - Visuals (e.g., beakers, graduated cylinders, balance scales, etc.) should be utilized to support reasoning. Each apple is about 70 g. If you purchase 4 apples, how many grams of apples will you purchase? 	- Use measurement tools and visuals to show how quantities combine, separate, or repeat in measurement contexts. - Use appropriate vocabulary when estimating, measuring, explaining their process, and describing attributes of objects. Students should be encouraged to use appropriate units when communicating, verbally and in writing. - Engage in problem contexts that involve price per unit scenarios (e.g., Each 1 L bottle costs \$2.00. What is the cost of 5 bottles?). \$

Previously 3.MD.A.2

3.GR.B REASON WITH SHAPES AND THEIR ATTRIBUTES.

3.GR.B.5: Explain that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category of shapes (e.g., quadrilaterals).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Identify and draw shapes having specific attributes (2.GR.A.5). Where are students going? - Classify two-dimensional figures based on the presence or absence of parallel lines, perpendicular lines, or angles (4.GR.C.9).	- Provide multiple examples to support students in recognizing that a shape can belong to multiple categories. For example, a square also belongs to the rectangle, rhombus and quadrilateral categories. - Students should be able to recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. - Use diagrams to show overlapping categories. For example, a Venn diagram can be used to show the relationship between quadrilaterals, rectangles, rhombuses, and squares.  - Use the inclusive definition for trapezoid (a quadrilateral with at least one pair of parallel sides).	- Physically group shapes by shared attributes (e.g., four sides, right angles) and discuss the larger category of shapes that can be used to describe the group. - Explore examples and non-examples to deepen understanding of categorizing shapes based on attributes. - Justify why shapes belong in certain categories. - Use precise vocabulary (e.g., side, equal sides, quadrilateral, etc.) when explaining relationship between shapes or groups of shapes (SMP 6). Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Geometric Reasoning & Measurement milestones in the Classifying Figures strand of the Numeracy Development Framework.

Previously 3.G.A.1

3.GR.C UNDERSTAND CONCEPTS OF AREA AND PERIMETER.

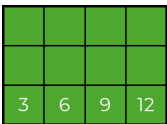
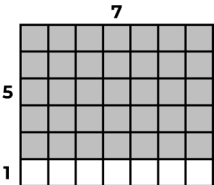
3.GR.C.6: Identify area as an attribute of a two-dimensional figure and measure area of rectangular space by counting square units (non-standard equal-sized units, square centimeters, square meters, square inches, square feet).

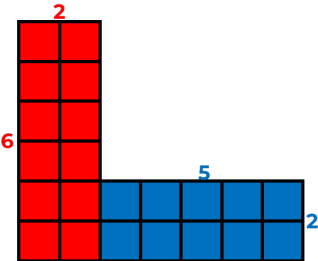
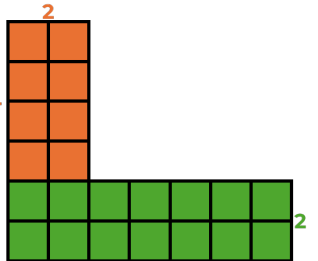
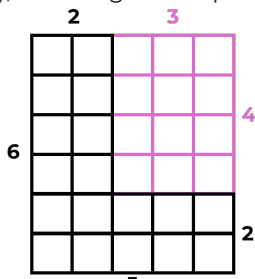
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Measure lengths using standard units (2.GR.A.1). Where are students going? - Apply the area and perimeter formulas for rectangles in context, including rectangles with missing dimensions 4.GR.A.3). - Relate the area of a triangle to the area of a rectangle using decomposition and visual models (4.GR.A.4).	- Area is the measure of space inside a figure or region and is measured in square units. - Area of a rectangular space can be found by counting square units or by multiplying side lengths. This standard is focused on counting square units to determine area of a figure. - Provide examples to support students in recognizing the difference between linear and square units. Linear units are one-dimensional and measure the length or height of an object. Square units are two-dimensional and measure space. Discuss how square units are different from linear units. linear unit — square unit - When using square units to measure the area of a region, students will cover the space with equal-sized square units (e.g., square-inch tiles, sticky notes) without gaps or overlaps. Then, count the number of square units to determine the area. The area of the rectangle is 8 square units. - Present area tasks in authentic contexts (e.g., painting a wall, flooring for a room).	- Engage in hands-on experiences to develop an understanding of area. Students should use square tiles (or other equal sized square units) to cover physical spaces and count square units to identify the area. - Count by ones or skip count when counting square units to measure area. Students should begin counting by ones. As they become more proficient, students can transition to skip counting of square units as a more efficient strategy. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 5 10 15 - Practice using appropriate units when describing and recording area (e.g., square centimeters, square inches, square units). - Discuss and explain the difference between area and perimeter. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Geometric Reasoning & Measurement milestones in the Reasoning with Geometric Structure strand of the Numeracy Development Framework.

Previously 3.MD.C.6

3.GR.C.7: Relate area to multiplication and addition.

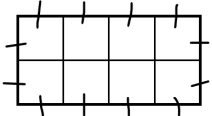
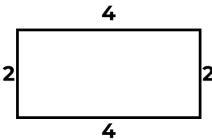
- Determine the area of a rectangle with whole-number side lengths by tiling and connecting the visual representation to multiplication of the side lengths to solve problems in context.
- Use tiling and area models to show that the area of a rectangle with whole number side lengths b and $c + d$ is the sum of $b \times c$ and $b \times d$, representing the Distributive Property of Multiplication.
- Determine the area of composite rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas.

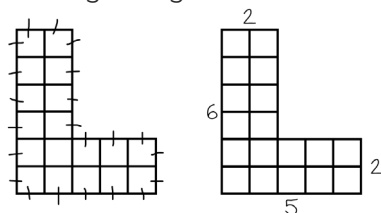
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Measure length using standard units (2.GR.A.1). Where are students going? - Apply area formulas for rectangles in context, including rectangles with missing dimensions (4.GR.A.3). - Relate area of triangle to the area of a rectangle (4.GR.A.4).	- Area is the measure of space inside a figure or region and is measured in square units. - Area can be found by counting square units or by multiplying side lengths. 3.GR.C.7a: Facilitate exploration to allow students to make connections between tiling and multiplication. Provide multiple examples to support students in making the generalization. As students begin to generalize, they apply multiplication to calculate area of rectangles without counting square units or tiling.  3 rows of 4 tiles 3 groups of 4 tiles 3×4  3.GR.C.7b: An area can be decomposed into smaller areas and then recomposed to find total area. For example, the rectangle below is decomposed into two smaller rectangles, a 5 by 7 rectangle and a 1 by 7 rectangle. A student can find the area of each rectangle and then add the areas to find the total area.  $6 \times 7 = (5 \times 7) + (1 \times 7)$ 3.GR.C.7c: A composite rectilinear figure is a figure made up of two or more rectangles joined together and sharing sides and corners. <i>Continued on next page</i>	- Engage in tiling of various rectangles in order to identify the relationship between area and multiplication. - Use visual and symbolic representations to explain how to determine the area of a rectangle. - Develop their proficiency in calculating area by initially tiling rectangles to calculate area. As students become more proficient, they will transition to using multiplication to calculate areas without needed visual square units or tiles. - Use appropriate units when describing and recording area (e.g., square centimeters, square inches, square units). - Use area models flexibly, decomposing rectangles in different ways to calculate partial and total areas of the rectangles. Encourage students to discuss and compare their results. - Use visual and symbolic representations to explain how to determine the area of a composite figure. - Decompose composite figures flexibly. Students can decompose composite figures in different ways to calculate partial and total areas. Encourage students to discuss and compare their results. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Geometric Reasoning & Measurement milestones in the Reasoning with Geometric Structure strand of the Numeracy Development Framework.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	3.GR.C.7c: Composite figures can be decomposed in multiple ways to calculate area. In the example below, the student decomposed the figure into two different rectangles and added each of the smaller areas to find the total area.  $(6 \times 2) + (2 \times 5) = 22$ square units  $(4 \times 2) + (2 \times 7) = 22$ square units 3.GR.C.7c: Students may notice that removing part of a rectangle can create a composite figure. For example, if the 4×5 rectangle (purple) is removed from the 6×5 rectangle, the composite figure is formed (black). To find the area, calculate the area of the large rectangle (6×5) and subtract the area of the removed rectangle (4×3), resulting in 18 square units.  $(6 \times 5) - (4 \times 3) = 18$ square units - Model the use of parentheses in equations so students are familiar with notation. However, students are not required to use parenthesis when writing equations. - Present area tasks in authentic contexts (e.g., painting a wall, flooring for a room).	

Previously 3.MD.C.7

3.GR.C.8: Identify perimeter as an attribute of two-dimensional figures and measure the distance around the outside of a figure by counting units or adding lengths.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Measure length using standard units (2.GR.A.1). Where are students going? - Apply perimeter formulas for rectangles in context, including rectangles with missing dimensions (4.GR.A.3).	- Perimeter is the measure around a figure or region and is measured in units. - Perimeter can be found by counting units or by using addition to add the lengths of each side of the figure. - Provide examples to support students in recognizing the difference between linear and square units. Linear units are one-dimensional and measure the length or height of an object. Square units are two-dimensional and measure space. Discuss how square units are different from linear units. linear unit — square unit  - Grade 3 students begin by counting linear units to determine perimeter. Model and encourage students to use marking to track their counting.  The perimeter of the rectangle is 12 units. - Facilitate exploration to allow students to make connections between counting units of individual side lengths and addition. Provide multiple examples to support students in making the generalization. As students begin to generalize, they apply addition to calculate perimeter of figures without counting units.  $2 + 4 + 2 + 4 = 12$ units The perimeter of the rectangle is 12 units. <i>Continued on next page</i>	- Engage in physical experiences to develop an understanding of perimeter. For example, students may walk around the perimeter of the playground or put string around the perimeter of a desk. - Use geoboards to create and find perimeter of various figures. Students can record their figures and perimeters using grid paper. - Develop their proficiency in determining perimeter by initially counting units. As students become more proficient, they will transition to using addition to calculate perimeters without needing to count. - Use appropriate units when describing and recording perimeter (e.g., centimeters, inches, units). - Use visual and symbolic representations to explain how to determine the perimeter of a rectangle or irregular figure. - Apply their understanding of attributes of rectangles to explain how you can calculate the perimeter of a rectangle when only two dimensions are given (3.GR.B.5). - Discuss and explain the difference between area and perimeter. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Geometric Reasoning & Measurement milestones in the Reasoning with Geometric Structure strand of the Numeracy Development Framework.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	- Students will apply these strategies to measure perimeter of irregular figures.  The perimeter of the figure is 15 units. - Present perimeter tasks in authentic contexts (e.g., fencing,).	

Adopted 2025

3.GR.C.9: Solve problems in contexts involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and finding rectangles with the same perimeter and different areas or with the same area and different perimeters.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Measure length using standard units (2.GR.A.1). Where are students going? - Apply perimeter formulas for rectangles in context, including rectangles with missing dimensions (4.GR.A.3).	- Perimeter is the measure around a figure or region and is measured in units. It can be found by counting units or by using addition to add the lengths of each side of the figure. - Area is the measure of space inside a figure or region and is measured in square units. It can be found by counting square units or by multiplying side lengths. - Present area and perimeter tasks in authentic contexts. Refer to the examples below. Find perimeter given side lengths: A rectangle playground is 6 meters long and 4 meters wide. Find the perimeter of the playground. Find unknown side lengths given perimeter: A rectangular garden has a perimeter of 24 feet. If one side of the garden is 8 feet, what are the lengths of the other sides? Find rectangles with same perimeter and different areas: Design two picture frames with the same perimeter but different interior areas. Finding rectangles with same area and different perimeters: Compare the size of the dog pens. Dog Pen A is 6 feet by 4 feet. Dog Pen B is 12 feet by 2 feet. How do their perimeters and areas differ? Which would be better for a dog and why?	- Create visual and symbolic representations to represent contexts. - Use visual and symbolic representations to explain how to determine the perimeter or unknown side lengths. - Create and compare rectangles with the same perimeter but different areas or same area and different perimeters. Explain why the perimeters or areas differ. - Use tables to record and analyze relationships between length, width, area, and perimeter of rectangles. - Apply understanding of attributes of rectangles to explain how to find unknown side lengths. For example, if a rectangle has three known sides (5 cm, 5 cm, 8 cm), reason that the fourth side must be 8 cm because opposite sides of a rectangle are equal (3.GR.B.5). - Discuss and explain the difference between area and perimeter. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Geometric Reasoning & Measurement milestones in the Reasoning with Geometric Structure strand of the Numeracy Development Framework.

Previously 3.MD.D.8

Reasoning with Data, Statistics, and Probability (DS)

Reasoning with Data, Statistics, and Probability (DS) Clusters	Connections to the Numeracy Development Framework
3.DS.A Represent and interpret data.	Standards within this cluster directly contribute to the Grade 3 Reasoning with Data, Statistics, and Probability milestones within the Collecting & Representing Data and Interpreting & Analyzing Data strands of the Numeracy Development Framework.

***Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

****Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.

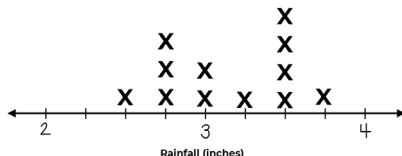
3.DS.A REPRESENT AND INTERPRET DATA.

3.DS.A.1: Ask and answer questions by collecting, organizing and summarizing data, recognizing the importance of context when analyzing data.

- Create a scaled data visualization (e.g., picture graph, bar graph for categorical data; line plot with fraction units of halves and fourths for numerical data) to display collected or given data.
- Summarize data presented in scaled data visualizations (e.g., picture graph, bar graph for categorical data; line plot for numerical data) by describing the story the data is telling.
- Identify patterns in the data and evaluate whether these patterns might apply to different contexts or situations.

Content Progression	Examples and Clarifications	Students should have the opportunity to...						
Where are students coming from? - Ask and answer questions by collecting, organizing (in picture graphs, bar graphs, line plots), and summarizing data (2.DS.A.1). Where are students going? - Ask and answer questions by collecting, organizing (bar graphs, line plots with fractional units), and summarizing data. Compare and contrast different data visualizations (4.DS.A.1).	- Understanding data goes beyond making graphs—it involves engaging in the full statistical process: asking meaningful questions, collecting and organizing data, and interpreting results to make sense of the world. Authentic applications help students see data as a tool for inquiry and decision-making, not just representation. - Categorical data refers to data that can be sorted into groups or categories (e.g., transportation to school or choice for class field trip, etc.). - Numerical data refers to data that can be counted or measured and can be expressed as a number (e.g., heights of students, temperature). Numerical data can be represented as whole numbers and fractions. 3.DS.A.1a: A picture graph represents data using symbols. In the example below, the symbol represents 2 students, therefore half of a symbol would equal 1 student. Symbols on picture graphs can be scaled to 2 or 10 and students should understand what half a symbol represents. Ways We Get to School <table><tr><td>Bus Rider</td><td>■ ■ ■ ■ ■ ■ ■ ■</td></tr><tr><td>Car Rider</td><td>■ ■ ■</td></tr><tr><td>Walker</td><td>■ ■</td></tr></table> ■ represents 2 students	Bus Rider	■ ■ ■ ■ ■ ■ ■ ■	Car Rider	■ ■ ■	Walker	■ ■	- Create their own statistical questions based on their interests and develop a plan to collect data. - Interpret a data visualization and formulate the statistical question it could have aimed to answer. - Make justifications about the type of data visualization used to display a data set, understanding the various options and the limitations of each. - Connect picture graphs and bar graphs. Begin to create bar graphs using bars with countable units, so each piece of data is visible. As students become more proficient, they will create bar graphs without countable units and scales other than 1. Class Field Trip Options for Class Field Trip Class Field Trip Options for Class Field Trip - Apply their understanding of fractions as numbers on a number line (3.NOS.F.12) to create line plots with fraction units and interpret partial symbols on a picture graph.
Bus Rider	■ ■ ■ ■ ■ ■ ■ ■							
Car Rider	■ ■ ■							
Walker	■ ■							

Continued on next page

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	• 3.DS.A.1a: A bar graph represents data using bars and labeled axes. Scales should be reasonable and allow for manageable skip-counting. When using a scale other than 1, students will use estimation to represent data points that are not multiples of the scale. • 3.DS.A.1a: A line plot represents data using symbols on a number line. Students will use whole numbers and fractions including halves and fourths. Monthly Rainfall (in inches)  Rainfall (inches) • 3.DS.A.1b: Ask questions such as, “What story is the graph telling?” or “What question could the data presented in the graph be trying to answer?” to prompt students to summarize the data. For example, when analyzing the picture graph above, students ask to consider what question the data could be trying to answer or what the data could mean about where students live in relation to the school. ◦ Sample response: The question could be, “How do students get to school?” Since most students are bus riders and only 3 students are walkers, I think that means they do not live close to school. • 3.DS.A.1c: Students consider whether what they notice in a data set could reasonably occur in another situation (e.g., another class, another day, or another group). For example, if a survey of students in Garret County shows a particular pattern in favorite activities, students might consider whether a similar pattern could reasonably appear if students in Worcester County were surveyed. • Consider the reasonableness of the number of categories (categorical data) and number of data points when selecting or designing problems and tasks.	• Apply understanding of measurement to measure lengths of objects to the nearest half and quarter unit to collect and display numerical data (3.GR.A.1). • Interpret data visualization in terms of the original context or situation. Students should contextualize data to justify if results would be similar or different in a new context. • Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 3 Reasoning with Data & Statistics milestones in the Collecting & Representing Data and the Interpreting & Analyzing Data strands of the Numeracy Development Framework.

Previously 3.MD.B.3; 3.MD.B.4

References

Hulbert, E. T., Petit, M. M., Ebby, C. B., Cunningham, E. P., & Laird, R. E. (2023). *A focus on multiplication and division: Bringing mathematics education research to the classroom* (2nd ed.). Routledge.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2020). *Elementary and middle school mathematics: Teaching developmentally* (10th ed.). Pearson

Some graphics obtained from <https://www.flaticon.com>

Provide Feedback

Your feedback is essential in helping us understand how these resources are being used, their effectiveness in supporting instruction, and how they can be improved to better serve educators, students, and stakeholders across the state. If you are interested in providing feedback, please completed the [Feedback: PreK-8 Mathematics Standards Companion Guide](#) survey.

The survey includes general questions about your overall impressions of the companion guides, as well as opportunities to provide grade-level specific feedback. Your responses will inform future revisions and professional learning efforts to ensure the guides are clear, useful, and aligned with the needs of Maryland's diverse learning communities.