



Grade 5

Maryland College and Career Ready Standards for Mathematics

Companion Guide

Mathematics Branch

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Purpose

PURPOSE OF COMPANION GUIDE

This document is designed to support a shared understanding of what each mathematics standard means, how it connects to other content, and how it can be authentically enacted in classrooms. It does not replace your high-quality instructional materials (HQIM) but instead works alongside them so that educators have a common interpretation of the mathematical expectations we set for students. Regardless of the HQIM in use, these documents provide clarity about the essence and intentions of the standards and illustrate how those expectations can be realized through instruction.

DOCUMENT STRUCTURE

Each domain begins with a set of cluster connections to the Numeracy Development Framework (NDF). These connections identify the ways in which individual clusters either:

- **Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.
- **Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

For each content standard, the companion document provides:

- **Content Progression:** A description of the content that comes before and after that standard to highlight how it fits within a coherent learning trajectory.
- **Examples and Clarifications:** Explanations and expansions of the standard language that provide additional detail about its intent. These may include illustrative examples, as well as common missteps to avoid that can narrow or distort the mathematical authenticity of the standard.
- **Student Opportunities for Mathematical Experience:** Statements describing the kinds of reasoning, modeling, problem solving, and sense-making students should have the chance to engage in as part of learning the content of the standard. Where relevant, these are connected to the Standards for Mathematical Practice (SMPs). Not every SMP is explicitly called out, but all can be present in any standard depending on how instruction is designed. What is included are non-exhaustive examples.

Standards for Mathematical Practice

“The Standards for Mathematical Practice (SMPs) describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” ([Common Core State Standards Initiative](#)). The SMPs develop along a continuum as students progress mathematically, with each grade’s content providing distinct opportunities for students to engage in and demonstrate the practices at an appropriate level of sophistication.

The SMPs should be integrated with content and not taught separately. As students participate in these practices on a regular basis, they become part of the mathematically productive habits of mind that support deep understanding. Intentionally incorporating opportunities to develop the SMPs enhances instruction and enriches students’ mathematical engagement. Students who learn to use these eight mathematical practices as they engage with mathematical concepts and skills have a greater chance of developing conceptual understanding (Van de Walle, et al, 2014).

The table below highlights opportunities to develop the SMPs in general across the given grade band, while the companion guides for individual standards provide more specific connections to the content of each grade. Educators should utilize this resource to assist in planning for opportunities when the SMPs can be emphasized in daily instructional practice. To facilitate the incorporation of SMPs within instruction, notations have been made in the Companion Guide to indicate when a specific SMP may be applicable for a particular standard. These examples should not limit educators from making additional meaningful connections to SMPs where appropriate.

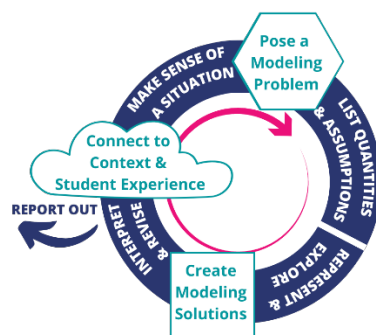
Standard for Mathematical Practice	What does it look like for Grades 3 – 5 students?	How can Grades 3 – 5 teachers support this practice?
Make sense of problems and persevere in solving them (SMP 1)	Students make sense of problems such as multi-step word problems, equal sharing, or comparing quantities using operations, area models, fraction strips, or other visual models. They identify what is being asked, plan an approach (e.g., draw a diagram, make a table), and track if it’s working. They revise when needed and use mathematical reasoning to determine if an answer is reasonable (e.g., “If I’m sharing 12 among 4 people, each should get more than 2”). Students increasingly view perseverance and revising strategies as part of doing math.	Teachers provide contextual problems aligned to grade-level content (operations, fractions, measurement) and build in time for students to plan their approach, often beginning with estimation or sense making. Students are encouraged to consider multiple solution paths and representations. When students get stuck, teachers prompt re-engagement (“What’s another way to show this?”) rather than redirect to correctness. Recognition is given not only for the correct answer but for strategic effort.

Standard for Mathematical Practice	What does it look like for Grades 3 – 5 students?	How can Grades 3 – 5 teachers support this practice?
Reason abstractly and quantitatively (SMP 2)	Students interpret problem situations using numbers, diagrams, and equations and attend to what each quantity represents (e.g., “12 is how many students, not how many buses”). They reason about relationships (e.g., understanding 3×4 as 3 groups of 4) and check whether representations make sense in context. Increasingly, students choose representations that help them reason about the situation, not just carry out a calculation.	Teachers offer opportunities for students to select appropriate representations (e.g., arrays, tables, diagrams, equations) and ask “What does this number or step tell us in the situation?” They facilitate flexible movement between context and representation, and emphasize that understanding quantities and their meaning supports effective reasoning before, during, and after calculation.
Construct viable arguments and critique the reasoning of others (SMP 3)	Students explain how and why their strategy works using models, numbers, words, or equations (e.g., “I multiplied because there were three equal groups”). They listen to other’s explanations, compare approaches, and ask clarifying questions to understand someone else’s reasoning. They begin to revise their own reasoning when a peer’s explanation reveals a clearer, more accurate, or more general way to make sense of the problem.	Teachers create structured opportunities for students to share and compare strategies and use prompts such as, “Can you explain why this works?” or “What makes this approach make sense?” They establish norms that emphasize reasoning and justification as tools for understanding and encourage students to listen to, respond to, and build on one another’s explanations in respectful, constructive ways.
Model with mathematics (SMP 4)	Students ask questions that can be explored mathematically (“How many do we need?”, “How long will it take?”, “What’s the change?”) and identify the key quantities in the situation that need to be represented. They form a mathematical representation (e.g., diagram, table, number line, graph, or equation) to analyze what is happening. They interpret what their model shows and adjust the model when something doesn’t fit or when another choice provides clearer insight. Their modeling cycle becomes define quantities → represent → analyze → interpret → adjust in an increasingly mindful way.	Teachers design tasks where mathematics helps illuminate a real or mathematical situation and prompt students to identify important quantities (“What do we need to keep track of?”). They support students in forming models that match the situation and encourage checking for alignment (“Does your model fit the scenario?”). Multiple models may be compared, but the emphasis stays on how the model improves understanding, not on producing a variety of representations.
Use appropriate tools strategically (SMP 5)	Students select tools, including diagrams, rulers, protractors, number lines, area models, tables, graphs, and digital tools, to help investigate situations or support reasoning. They begin choosing tools purposefully (“A table helps me see the pattern”), not just because the tool is familiar. They recognize when a chosen tool isn’t giving the insight they need and switch to another that better fits the task.	Teachers introduce a variety of tools and help students discover what each tool can reveal about a situation. They ask reflective questions such as “Which tool could help you see this more clearly?” or “What does this representation show that another does not?” Teachers encourage evaluating the strengths and limitations of different tools and ensure tools are used to support, not replace, mathematical reasoning.

Standard for Mathematical Practice	What does it look like for Grades 3 – 5 students?	How can Grades 3 – 5 teachers support this practice?
Attend to precision (SMP 6)	Students use increasingly accurate mathematical language, notation, and units to communicate their thinking. They label representations (e.g., diagrams, number lines, tables, bar graphs, and line plots) to show what quantities or categories mean. They check calculations for accuracy and reasonableness in context, including in data displays (“Does this point belong here?”). Students notice when imprecision in language, diagrams, labels, or data representations leads to confusion and revise their work to make their thinking clearer.	Teachers emphasize naming quantities, labeling representations (including axes and data categories), choosing appropriate scales, and using correct units. They prompt students to clarify what numbers, labels, and symbols represent (e.g., “ <i>What does this number describe?</i> ” or “ <i>What does this label tell the reader?</i> ”) and support students in checking that their representations and calculations are clear and accurate. Precision is reinforced as a habit of communicating mathematical ideas so others can understand them.
Look for and make use of structure (SMP 7)	Students use structure to understand operations more deeply. For example, seeing multiplication as equal groups or arrays, understanding division as sharing or grouping, and using place-value structure to support efficient reasoning. They apply structure to decompose numbers, expressions, and shapes (e.g., “ 24×6 is like $(20 \times 6) + (4 \times 6)$ ”), and they distinguish between additive and multiplicative situations. Students recognize when problems share a common structure and use that insight to choose strategies and explain why those strategies work.	Teachers emphasize structural relationships in arithmetic (arrays, area models, place-value relationships, and properties of operations). They guide students in articulating how the structure of numbers, shapes, or expressions supports a strategy and prompt them to contrast structurally different problem types. Representations are used to make underlying structure visible and to reinforce conceptual understanding rather than procedural repetition.
Look for and express regularity in repeated reasoning (SMP 8)	Students observe regularity in operations and repeated calculations. For example, noticing how products grow in predictable ways when one factor increases, or how repeated addition produces equal groups and eventually multiplication. They recognize stable place-value relationships (“Each shift to the left makes the value ten times larger”) and see how similar reasoning approaches reappear across different division or fraction problems. Students use these repeated behaviors to predict outcomes, check their work, or form generalizations about how operations behave.	Teachers highlight repeated reasoning in multiplicative situations, place-value patterns, fraction equivalence, and strategies that rely on predictable changes (e.g., adding another equal group or repeatedly partitioning a shape or set). They prompt students to describe what repeats across cases (“What changes in the number each time you shift one place to the left?”) and use that insight to make or test conjectures. Teachers encourage students to explain <i>why</i> a shortcut or generalization holds by connecting it to the repeated reasoning that produced it.

Modeling

Mathematical modeling is the process of using mathematics to interpret and make sense of the world. Developing and engaging with the modeling cycle is a critical part of building students' mathematical proficiency. As students progress through the grades, they take on greater responsibility in all aspects of the modeling cycle. They more independently formulate questions, define constraints, and distinguish between assumptions that can be made and facts that must be verified. They are provided growing opportunities to choose models and tools that help them explore a situation and uncover the underlying mathematics. They also become more adept at evaluating the validity of their solutions and revisiting earlier stages of the cycle when initial assumptions prove invalid or incomplete. Participation in the mathematical modeling cycle nurtures mathematical creativity, fosters productive discourse, encourages persistence when solutions are not straightforward, and supports the development of positive mathematics identity.



Mathematical modeling is a significant aspect of MCAP Grades 3-8. Modeling is assessed in alignment to grade-level content. Modeling items are both machine scored and constructed response. The constructed response items are scored using [MCAP Mathematics Modeling Holistic Rubric](#). A calculator will be available for all Modeling assessment items.

Opportunities to connect modeling with content standards are identified within the Companion Guide. These opportunities are entry points for integration, but they should not limit educators from making additional meaningful connections to modeling where appropriate.

Modeling Evidence Statements	What does this look like for students in Grades 3 – 5?
M.1 Formulate a question about a particular context that can be answered using mathematics.	Students examine real or imagined contexts and formulate questions that can be answered using mathematics. They move beyond general curiosity to identify specific quantities, comparisons, or relationships that can be measured, counted, or calculated. For example, given a situation about planning a school fundraiser, students might refine a broad question such as “Will we raise enough money?” into questions like “How much money will we raise if each student sells three items?” or “How many items need to be sold to reach the fundraising goal of \$500?”
M.2 Determine the information needed and what assumptions must be made to answer a mathematical question about a context.	Students examine a situation to decide what details matter when answering a mathematical question. They sort relevant from irrelevant information, identify what is already known and what still needs to be determined, and recognize when assumptions are necessary to proceed. At this grade band, assumptions are typically based on simplifying the context so it can be analyzed mathematically. For example, in a school fundraiser scenario, students may decide they need to know how many students are participating and the price of each item sold. They may assume that all items cost the same or that each student sells the same number of items to estimate the total amount raised.
M.3 Represent a context mathematically by selecting and applying appropriate concepts, tools, and representations to define variables, model relationships, and/or identify constraints.	Students organize information from a situation to show how quantities are connected. They use representations such as drawings, tables, diagrams, equations, and simple graphs to make relationships visible and support their thinking. As they work, students attend to limits or conditions in the situation that affect how the context can be represented. For example, when modeling a school fundraiser, students may create a table showing how many items are sold and the total money raised or write a relationship such as <i>total money = number of items × price</i> to represent how the quantities are related.
M.4 Use a mathematical model to perform computations, make predictions, or answer the question posed about a context.	Students engage with their models as tools for exploring how quantities in a situation are related and to support decisions/conclusions about the context. They use the model to compare scenarios, notice patterns, and test how changes to one part of the situation affects others. Their models are used to guide their thinking rather than used for a single calculation. For example, using a fundraiser model, students may explore how increasing the number of items sold changes the total amount raised and use that relationship to compare different sales scenarios or consider how adjusting the price per item would affect the total.

Modeling Evidence Statements	What does this look like for students in Grades 3 – 5?
M.5 Interpret the results or conclusions of a mathematical model in terms of the original context or situation.	Students reflect on what their model reveals about the situation and connect the outcomes of the model back to the original question. They explain what the numbers or comparisons produced by the model mean in context and what those results suggest about decisions or next steps. For example, after modeling a school fundraiser, students may explain whether the amount of money raised meets the goal, describe what comparisons between sales plans show, or explain what the model suggests about how changing price affects outcomes.
M.6 Critique the validity of a mathematical model and its conclusions. Revise assumptions, representations, or mathematical approaches based on the context, new understanding, or additional information.	Students examine whether a mathematical model and its conclusions make sense for the situation being studied. Through sharing their thinking and comparing models with peers, students consider whether assumptions were reasonable, whether important details were overlooked, and whether the model reflects realistic constraints of the context. Based on new understanding, feedback, or additional information, students revise their assumptions or adjust their model to better represent the situation. For example, after sharing fundraising models, students may notice that another group included a limit on how many items a student could reasonably sell. Recognizing that this constraint changes the total amount raised, they revise their assumptions and adjust the model before reconsidering their conclusions.

Reasoning

Students who become adept at reasoning in mathematics are better able to transfer their learning to new and novel contexts and build confidence in their mathematical abilities. Reasoning is both a foundational skill for students and a critical focus of instruction for mathematics educators. Opportunities to reason should be embedded throughout all grade levels and students must be invited to justify their solutions, use multiple representations and tools to support conclusions, and engage in meaningful reflection. They should frequently be called upon to evaluate their own reasoning and that of others, decide whether a solution path is valid, identify and correct errors, and strengthen arguments. As outlined below, mathematical reasoning is a cornerstone skill that should be integrated across all grade-level content.

Reasoning is a significant topic in the MCAP assessment. Reasoning is assessed in alignment to grade-level content. Reasoning items are both machine scored and constructed response. The constructed response items are scored using [MCAP Mathematics Reasoning Holistic Rubric](#). A calculator will be available for all Reasoning assessment items.

Opportunities to connect reasoning with content standards are identified within the Companion Guide. These opportunities are entry points for integration, but they should not limit educators from developing reasoning skills where appropriate.

Reasoning Evidence Statements	What does this look like for students in Grades 3 – 5?
R.1 Use visual, symbolic, or contextual representations to explain mathematical ideas.	Students use representations to make their thinking visible and to build a more complete understanding of mathematical ideas. They connect multiple representations to show how the same relationship or idea can be expressed in different ways, and they explain how one representation supports or clarifies another. For example, a student may use an area model and an equation together to explain why two multiplication strategies result in the same product or connect a fraction strip and a number line to show how fractions with different denominators represent the same value.
R.2 Identify and critique errors in reasoning, misapplied procedures, or invalid assumptions, and provide a mathematically valid alternative.	Students use mathematical discussion to evaluate whether reasoning is sound and conclusions are justified. By listening to peers' explanations and asking clarifying or challenging questions, students learn to recognize when a claim depends on an unstated assumption or only works in certain cases. Through these exchanges, students revise their explanations to better reflect the mathematical idea. For example, during a class discussion comparing fractions, a student may claim that $\frac{3}{8}$ is greater than $\frac{1}{2}$ because 3 is greater than 1. Another student questions this reasoning by pointing out that the pieces are different sizes and asks how the fractions compare when the same whole is considered. As the discussion continues, the class revises the explanation to reflect a more accurate comparison.

Reasoning Evidence Statements	What does this look like for students in Grades 3 – 5?
R.3 Construct coherent chains of reasoning that connect mathematical properties, ideas, and evidence, such as examples or counterexamples, to support or refute mathematical statements or claims.	Students support mathematical claims by explicitly linking ideas so that their reasoning is clear and convincing. They connect definitions, properties, observations, or examples to show how multiple ideas work together to justify a conclusion. Rather than presenting isolated statements or answers, students explain how their ideas are connected to support or refute a claim. For example, when explaining why two fractions are equivalent, a student may connect the idea that both fractions represent the same amount of a whole with the observation that the whole is partitioned differently but the shaded portions cover the same area. They use both ideas together to justify why the fractions are equal.
R.4 Identify and extend patterns or regularities to form general rules and justify their validity using mathematical reasoning.	Students use repeated reasoning to move from noticing patterns to forming general rules. The focus is not on predicting the next term in a sequence, but on explaining <i>why the same relationship holds across many cases</i>. Students identify what remains consistent and connect that regularity to underlying mathematical ideas. This supports them in justifying a rule that applies beyond individual examples. For example, a student may notice that multiplying a whole number by 10 always changes the value in the same way. The student explains that each digit represents ten times as much as the digit to its right, so multiplying by 10 shifts every digit one place to the left, which increases the value of the number by a factor of ten. By applying this reasoning to several numbers, the student justifies that the rule holds for all whole numbers.

Numeracy Development Framework

Grade 5 represents a consolidation and extension of the multiplicative and fractional reasoning developed across the elementary grades. Students work with greater precision and flexibility as they operate with whole numbers and fractions, represent relationships more explicitly, and reason about quantities across contexts. Across domains, students connect units, operations, and representations to interpret and solve problems, and prepare for more advanced reasoning about ratios, proportions, and functions in middle school.

Within Number & Operation Sense, students extend place value understanding to include decimals to the thousandths and continue working with whole numbers through millions. This expanded number system supports more precise comparison of quantities and strengthens students' use of estimation to judge the reasonableness of results. Students develop fluency with multiplication of multidigit whole numbers and use reasoning-based strategies for division. Fraction work continues to be a focal point as students add and subtract fractions with unlike denominators and multiply fractions by whole numbers and other fractions. This reinforces the idea that fractions are numbers that can be combined and operated on when expressed in compatible units.

In Algebraic Thinking, students represent relationships using numerical expressions and explicitly defined rules. They use grouping symbols to show how quantities are related and to support correct interpretation of more complex operations. Students generate input-output pairs from rules and represent these relationships as ordered pairs. This strengthens their ability to describe how one quantity changes in relation to another and to begin formalizing their functional thinking.

Geometric Reasoning and Measurement deepens students' understanding of units and spatial relationships. Students classify quadrilaterals and triangles based on angle measures and side lengths and convert between units within the same measurement system. Work with volume extends multiplicative reasoning as students determine the volume of rectangular prisms by reasoning about how layers of equal-sized units fill space. Students also use the coordinate system to locate points, which connects spatial reasoning with numerical representation.

In Reasoning with Data, Statistics, and Probability, students move beyond organizing data to evaluating and interpreting it. They examine whether data visualizations accurately represent the data, interpret distributions using features such as mode, range, and gaps, and determine whether general statements about what usually happens or how often something occurs are reasonable given the data and the context. This work marks the beginning of probabilistic reasoning, as students use data to make and justify claims about likelihood and typical outcomes. It strengthens their ability to analyze information and justify conclusions based on evidence.

The developmental milestones identified in the Numeracy Development Framework for Grade 5 reflect a unification of multiplicative reasoning, fractional understanding, and relational thinking across domains. This companion document highlights how these ideas align with Grade 5 standards and identifies the understandings most critical to monitor as students prepare for increasingly sophisticated reasoning about relationships and quantities in middle school.

Number and Operation Sense (NOS)

Number and Operation Sense (NOS) Clusters	Connections to the Numeracy Development Framework
5.NOS.A Understand place value system.	Standards within this cluster directly contribute* to the Grade 5 Number and Operation Sense milestones within the Numbers & Number Sets strand of the Numeracy Development Framework.
5.NOS.B Perform operations on multi-digit whole numbers and with decimals to hundredths.	Standards within this cluster directly contribute* to the Grade 5 Number and Operation Sense milestones within the Operations and Estimation strands of the Numeracy Development Framework.
5.NOS.C Use equivalent fractions as a strategy to add and subtract fractions.	Standards within this cluster directly contribute* to the Grade 5 Number and Operation Sense milestones within the Fractions & Decimals strand of the Numeracy Development Framework.
5.NOS.D Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	Standards within this cluster directly contribute* to the Grade 5 Number and Operation Sense milestones within the Fractions & Decimals strand of the Numeracy Development Framework.

***Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

****Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.

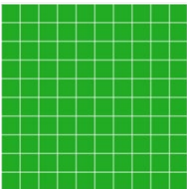
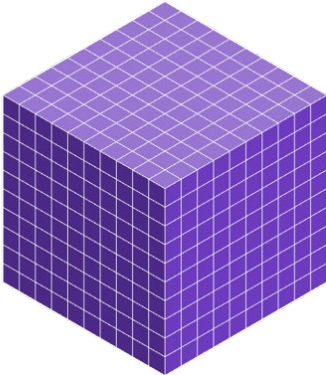
5.NOS.A UNDERSTAND THE PLACE VALUE SYSTEM.

5.NOS.A.1: Explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Explain that a digit in one place represents ten times what it represents in the place to its right (4.NOS.A.2). Where are students going? - Explain patterns when multiplying whole numbers and decimals by 10 (5.NOS.A.2). - Read, write, and compare decimals to thousandths (5.NOS.A.3).	- Students apply their understanding of the value of a digit based on place value to relate the value of digits in different places within a number. - A 7 in the thousands place represents 7,000. However, when 7 is in the hundreds place, it represents 700, which is one-tenth of 700. This relationship can also be represented with equations. $\begin{array}{l} 7000 \div 10 = 700 \\ 700 \div 10 = 70 \\ 70 \div 10 = 7 \\ 7 \div 10 = 0.7 \end{array} \quad \begin{array}{l} \frac{1}{10} \times 7000 = 700 \\ \frac{1}{10} \times 700 = 70 \\ \frac{1}{10} \times 70 = 7 \\ \frac{1}{10} \times 7 = 0.7 \end{array}$ - Focus on the value of digits and use precise language (e.g., “ten times greater,” “divided by 10,” “one-tenth of”). Avoid teaching rules like “add a zero to the value of a digit when moving left,” as these can lead to misconceptions, especially as students work with decimals.	- Compare the values of digits appearing in different place values to make sense of and explain the relationship. - Explore number sequences (e.g., 0.03, 0.3, 3, 30, 300, 3000) to identify patterns and the relationship between place values. Students should also form rules that can be applied to extend the pattern. - Apply their understanding of the base-ten structure to explain why a digit in one place is ten times the value of the same digit in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left (SMP 7). - Apply their understanding of multi-digit whole numbers to explain the relationship between place values of decimals to the thousandths place. For example, a students may explain that the 8 in 4.816 is worth ten times more than the 8 in 2.083.

Previously 5.NBT.A.1

5.NOS.A.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10 and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.

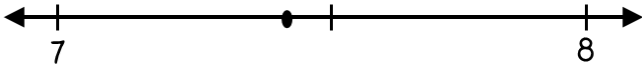
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right (4.NOS.A.2) and $\frac{1}{10}$ of what it represents in the place to its left (5.NOS.A.1). Where are students going? - Write numerical expressions involving whole number exponents (6.AT.B.5).	- Connect equations to visual models to help students develop understanding of a power of 10. $10 \times 10 = 100$  $10 \times 10 \times 10 = 10 \times 100 = 1000$  - Use equations sets to explore patterns in the number of zeros of the product when multiplying by a power of 10. Prompt students to identify and describe patterns. Make connections between the number of zeros and the place value (e.g., each equation shows a place value that is ten times greater). $10 \times 10 = 100$ $10 \times 10 \times 10 = 10 \times 100 = 1,000$ $10 \times 10 \times 10 \times 10 = 10 \times 1,000 = 10,000$ - Powers of 10 can also be expressed using exponents, written as 10^n, where n is a whole number exponent. $10^1 = 10$ $10^2 = 10 \times 10 = 100$ $10^3 = 10 \times 100 = 1,000$ $10^4 = 10 \times 1000 = 10,000$ <i>Continued on next page</i>	- Use visual models and equations to explore multiplication of whole numbers by powers of 10. When working with decimals, students should make connections to money amounts. - Explore patterns with whole numbers. As students develop an understanding, they can apply their understanding to explore patterns with decimals. - Describe patterns and justify their validity using mathematical reasoning. - Record powers of 10 using exponents.

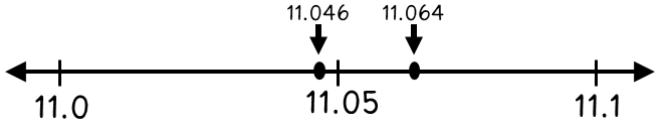
Content Progression	Examples and Clarifications	Students should have the opportunity to...
	- Provide additional examples involving decimals to support students in extending their understanding to decimal numbers. Relate decimal numbers to money amounts to support students in making connections. $10 \times \\$10 = \\100.00 $10 \times \\$1 = \\10.00 $10 \times \\$0.10 = \\1.00 $10 \times \\$0.01 = \\0.10 - Focus on the value of numbers and use precise language. Avoid teaching rules like “add a zero to the end of the number” as these can lead to misconceptions, especially as students work with decimals.	

Previously 5.NBT.A.2

5.NOS.A.3: Read, write, and compare decimals to thousandths.

- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form (e.g. $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$).**
- Estimate decimal quantities by reasoning about their location on a number line, their relationship to benchmark numbers (e.g. 0, 0.25, 0.5, 0.75, 1) and rounding to any place.**
- Compare two decimals to thousandths by reasoning about the values of the digits and the location on the number line. Record the results of comparisons with the symbols $>$, $=$, $<$.**

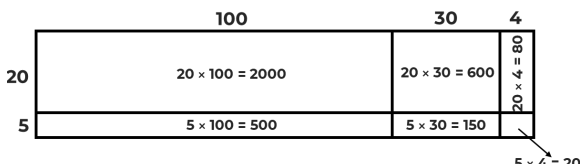
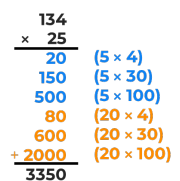
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Apply and extend place value understanding to multi-digit whole numbers within 1,000,000 (4.NOS.A.1). Where are students going? - Fluently multiply and divide multi-digit decimals to the thousandths (6.NOS.B.3). - Represent a rational number as a point on a number line (6.NOS.C.6).	- Model correct reading of whole numbers and decimals. For example, 137.621 is read as “one hundred thirty-seven and six hundred twenty-one thousandths,” not “one hundred thirty-seven point six two one.” The word “and” should be used only to separate the whole number from the decimal part. It should not be used when naming whole numbers (e.g., “one hundred thirty-seven,” not “one hundred and thirty-seven”). 5.NOS.A.3a: Use base-ten numerals (42.536), number names (forty-two and five hundred thirty-six thousandths), and expanded form ($40,000 + 2,000 + 5 \times (\frac{1}{10}) + 3 \times (\frac{1}{100}) + 6 \times (\frac{1}{1000})$) flexibly. 5.NOS.A.3b: Estimation is flexible. The number line shown helps a student to visualize 7.413 as a number between 7 and 8 but that is also close to 7.5. Using a benchmark number like 7.5 may be more useful in certain contexts, while rounding to 7 may be useful in others.  5.NOS.A.3b: When rounding, consider the value of the number and its relationship to benchmarks. For example, 7.413 rounded to the nearest whole number is 7 because 7.413 is closer to 7 as shown above. Avoid using tricks (e.g., “five and above, give it a shove”) without connecting to number relationships.	- Use a variety of representations to read, write, and visualize decimals, making connections between the representations. - Think flexibly about decimals and quantities. Estimation should be frequently embedded into discussions about decimals and their relationships. - Use number lines to justify and explain how to round a given decimals to the nearest hundredth, tenth, or whole number or to compare a set of decimals. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Number and Operation Sense milestones in the Numbers & Number Sets strand of the Numeracy Development Framework.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	• 5.NOS.A.3c: Use number lines to visualize the relationship between numbers to compare. The number line shows that $11.064 > 11.046$ because 11.064 is further to the right.  A horizontal number line with arrows at both ends. It has three major tick marks labeled 11.0, 11.05, and 11.1. Between 11.0 and 11.05, there are two smaller tick marks. An arrow points from the label '11.046' to the first of these smaller tick marks. Another arrow points from the label '11.064' to the second of these smaller tick marks.	

Previously 5.NBT.A.3; 5.NBT.A.4

5.NOS.B PERFORM OPERATIONS ON MULTI-DIGIT WHOLE NUMBERS AND WITH DECIMALS TO THOUSANDTHS.**5.NOS.B.4: Fluently multiply multi-digit whole numbers.**

- Apply estimation strategies to estimate products.
- Use computational strategies (e.g., partial products, doubling and halving) efficiently to multiply.
- Use a standard algorithm to multiply.
- Determine and explain when a strategy or algorithm is most efficient.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers (4.NOS.B.5). Where are students going? - Fluently multiply and divide decimals to the thousandths (6.NOS.B.3).	- Model various approaches and encourage flexibility. Avoid modeling or requiring a single approach. 5.NOS.B.4b: Apply the Distributive Property of Multiplication to use partial product strategy. Decompose factors by place value and then multiply. Then add the partial products to find the total. For example, a student may show the steps shown below to solve 25×134.   5.NOS.B.4b: Students apply the Associative Property of Multiplication to use the doubling and halving strategy. For example, to solve 12×325, a student may half one factor and double the other factor. $12 \times 325 = (6 \times 2) \times 325 = 6 \times (2 \times 325) = 6 \times 650$ 	- Make explicit connections between different approaches. - Use strategies flexibly, making decisions on which strategy or method to use based on the numbers or context of the given problem (SMP 5). - Explain their reason for selecting a particular strategy. For example, a student may explain “when solving 12×325, I used the doubling and halving strategy because I could easily halve 6 and double 325. Then, I will have a 1-digit by 3-digit problem to solve which is easier.” - Engage in Number Talks to promote flexible thinking and provide opportunities to explain their thinking and make connections to other strategies. - Apply estimation strategies to assess the reasonableness of products. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Number and Operation Sense milestones in the Operations and Estimation strands of the Numeracy Development Framework.

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Content Progression	Examples and Clarifications	Students should have the opportunity to...
	5.NOS.B.4c: When introducing the standard algorithm, make connections to other approaches (e.g., partial products). $\begin{array}{r} 134 \\ \times 25 \\ \hline 20 \\ 150 \\ 500 \\ 80 \\ 600 \\ + 2000 \\ \hline 3350 \end{array}$ $\begin{array}{r} 134 \\ \times 25 \\ \hline 670 \\ + 2680 \\ \hline 3350 \end{array}$ 5.NOS.B.4d: When determining efficiency, consider the quantities, their relationships to benchmark numbers, and how those relationships influence strategy choice. Use think-alouds to model this decision-making process. For example, when solving 499×56, it may be more efficient to solve $(500 \times 56) - (1 \times 56)$ than using the standard algorithm.	

Previously 5.NBT.B.5

5.NOS.B.5: Divide multi-digit whole numbers with up to four-digit dividends and two-digit divisors.

- Apply estimation strategies to estimate quotients.
- Use partial quotients efficiently to divide.
- Use properties of operations and/or the inverse relationship between multiplication and division to divide.
- Represent and explain the computation by connecting rectangular arrays, area models, and/or equations to the meaning of division.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Divide whole numbers with up to four-digit dividends and one-digit divisors with and without remainders (4.NOS.B.6). Where are students going? - Fluently divide multi-digit numbers (6.NOS.B.2).	- Students continue to build understanding of division through a balance of concrete, semi-concrete, and abstract approaches. Model various approaches and encourage flexibility. Avoid modeling or requiring a single approach. 5.NOS.B.5b: Partial quotients involves subtracting groups of the divisor from the dividend to make computation more manageable. For example, a student may show the steps shown below to solve $1,256 \div 12$. $\begin{array}{r} 22 \\ 12 \overline{)1256} \\ \underline{-1200} \quad (12 \times 100 = 1200) \\ 56 \\ \underline{-24} \quad (12 \times 2 = 24) \\ 32 \\ \underline{-24} \quad (12 \times 2 = 24) \\ 8 \end{array}$ Make connections to rectangular arrays and/or area models as students develop proficiency with using partial quotients.	- Make explicit connections between different approaches. - Develop proficiency with division strategies. Students may begin exploring strategies to divide 2-digit dividends by one-digit divisors. As students become more proficient, they will apply their understanding of these strategies to 3- and 4-digit dividends. - Use strategies flexibly, making decisions on which strategy or method to use based on the numbers or context of the given problem (SMP 5). - Engage in Number Talks to promote flexible thinking and provide opportunities to explain their thinking and make connections to other strategies. - Apply estimation strategies to assess the reasonableness of quotients. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Number and Operation Sense milestones in the Operations and Estimation strands of the Numeracy Development Framework.

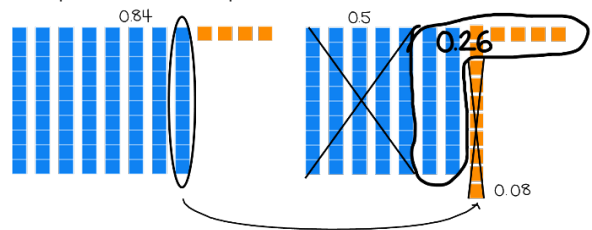
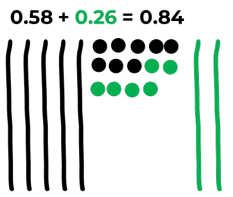
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Content Progression	Examples and Clarifications	Students should have the opportunity to...
	• 5.NOS.B.5c: Students apply their understanding of multiplication and division as inverse operations to think of a division problem as multiplication problem with a missing factor. For example, when solving $6,496 \div 32$, a student can think of $32 \times ? = 6,496$. The student can use known facts to find the missing factor, $32 \times 200 = 6,400$ and $32 \times 2 = 64$ so $6,496 \div 32 = 203$. • 5.NOS.B.5d: Students explain how their representation or strategy accurately represents division. For example, if using partial quotients to solve, a student may explain that equal group of 12 were subtracted from the total to determine how many total groups of 12 would equal 1,256.	

Previously 5.NBT.B.6

5.NOS.B.6: Apply and extend previous understanding of addition and subtraction of whole numbers to add and subtract decimals to hundredths.

- Apply estimation strategies to estimate sums and differences of decimals.
- Use concrete models, drawings, strategies based on place value (e.g., partial sums), properties of operations and/or the inverse relationship between addition and subtraction to add and subtract decimals.
- Use an algorithm to add and subtract decimals.
- Determine and explain when a strategy or algorithm is most efficient.

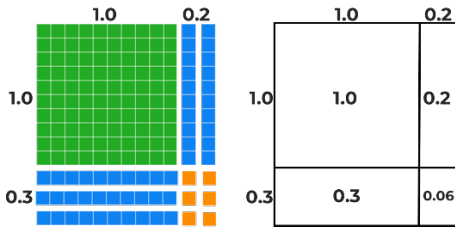
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Fluently add and subtract multi-digit whole numbers within 1,000,000 (4.NOS.B.3). Where are students going? - Fluently multiply and divide multi-digit decimals to the thousandths (6.NOS.B.3).	- Students develop an understanding of addition and subtraction of decimals through a balance of concrete, semi-concrete, and abstract representations. Avoid moving to abstract approaches too quickly without ensuring students have sufficient experience with concrete and semi-concrete approaches. 5.NOS.B.6b: Use concrete models, drawings or grid paper to add and subtract decimals. Support students in making connections between models to promote efficiency. For example, a student may use base ten manipulatives to represent and solve $0.84 - 0.58$.  5.NOS.B.6b: Apply understanding of addition and subtraction as inverse operations to think of a subtraction problem as an addition problem with a missing addend. For example, when solving $0.84 - 0.58$, a student may think of $0.58 + ? = 0.84$.  <i>Continued on next page</i>	- Make explicit connections between different approaches. - Develop proficiency with strategies for adding and subtracting decimals. Students may begin working with tenths. As students become more proficient, they will apply their understanding of these strategies to add and subtract decimals to hundredths. - Apply understanding of equivalent decimals appropriately align whole numbers and decimals when using the standard algorithm to add or subtract. - Use strategies flexibly, making decisions on which strategy or method to use based on the numbers or context of the given problem (SMP 5). - Explain their reason for selecting a particular strategy. For example, a student may explain “When solving $0.94 + 1.86$, I used the standard algorithm because the problem will require regroup in multiple places,” or “When solving $0.24 + 0.63$, I mentally used partial sums since regrouping wasn’t needed.” - Engage in Number Talks to promote flexible thinking and provide opportunities to explain their thinking and make connections to other strategies. - Apply estimation strategies to assess the reasonableness of sums or differences. - Apply addition and subtraction strategies to solve problems in context involving decimals.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	5.NOS.B.6c: When introducing the standard algorithm, make connections to other approaches (e.g., partial sums). $ \begin{array}{r} 1.35 \\ + 0.47 \\ \hline 0.12 \\ 0.7 \\ + 1.0 \\ \hline 1.82 \end{array} \qquad \begin{array}{r} 1 \\ 1.35 \\ + 0.47 \\ \hline 1.82 \end{array} $ 5.NOS.B.6c: When adding or subtracting decimals with different number of decimals places (e.g., $2.68 - 1.3$), emphasize that equivalent decimals are used to maintain place value alignment (e.g., 1.3 is rewritten as 1.30 because $1.3 = 1.30$). Avoid directing students to “add a zero” as this is mathematically inaccurate and does not focus on meaning of the numbers or place value. $ \begin{array}{r} 2.68 \\ - 1.30 \\ \hline 1.38 \end{array} $ 5.NOS.B.6d: When determining efficiency, consider the quantities, their relationships to benchmark numbers, and how those relationships influence strategy choice. Use think-alouds to model this decision-making process.	

Previously 5.NBT.B.7

5.NOS.B.7: Apply and extend previous understanding of multiplication and division of whole numbers to multiply and divide decimals to hundredths in context.

- Apply estimation strategies to estimate products and quotients.
- Use concrete models, drawings, and arrays to multiply and divide.
- Use strategies based on place value (e.g., partial products, partial quotients) to multiply and divide.
- Use properties of operations and/or the inverse relationship between multiplication and division to multiply and divide.
- Represent and explain the computation by connecting concrete models, drawings, and/or equations to the meaning of multiplication and division.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers (4.NOS.B.5). - Fluently multiply multi-digit whole numbers (5.NOS.B.4). Where are students going? - Evaluate expressions in which letters stand for numbers (6.AT.B.6). - Add and subtract rational numbers using multiple strategies and representations. (7.NOS.A.1).	- When multiplying, the product should not have any non-zero digits beyond thousandths. - Students develop an understanding of multiplication and division of decimals through a balance of concrete, semi-concrete, and abstract representations. Avoid moving to abstract approaches too quickly without ensuring students have sufficient experience with concrete and semi-concrete approaches. 5.NOS.B.7b: Begin using concrete models to multiply. Support students in making connections between concrete models and visual models, such as the area model, to promote efficiency. For example, the place value model (left) and area model (right) represent the expression, 1.3×1.2.  Discuss the limitations of using base-ten manipulatives for multiplying decimals. Students often use flats, rods, and units to represent ones, tenths, and hundredths respectively. However, this model cannot represent thousandths, which are often needed when multiplying decimals to hundredths. <i>Continued on next page</i>	- Identify and describe patterns in place value when multiplying decimals. For example, a student may notice and explain that when tenths are multiplied by tenths the product is to hundredths and when tenths are multiplied by hundredths the product is to thousandths. - Make explicit connections between different approaches. - Develop proficiency with strategies for multiplying and dividing decimals. Students may begin working with tenths. As students become more proficient, they will apply their understanding of these strategies to multiply and divide decimals to hundredths. - Use strategies flexibly, making decisions on which strategy or method to use based on the numbers or context of the given problem (SMP 5). - Engage in Number Talks to promote flexible thinking and provide opportunities to explain their thinking and make connections to other strategies. - Apply estimation strategies to assess the reasonableness of products and quotients. - Discuss what happens to the product (or quotient) when multiplying (or dividing) decimals greater than and less than 1. - Apply multiplication and division strategies to solve problems in context involving decimals.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	5.NOS.B.7c: Partial products involves decomposing factors by place value, multiplying the partial factors, and then adding the partial products to find the total. For example, a student may show the steps shown below to solve 1.72×2.4. $\begin{array}{r} 1.72 \\ \times 2.4 \\ \hline 0.008 \quad (0.4 \times 0.02) \\ 0.280 \quad (0.4 \times 0.7) \\ 0.400 \quad (0.4 \times 1) \\ 0.040 \quad (2 \times 0.02) \\ 1.400 \quad (2 \times 0.7) \\ + 2.000 \quad (2 \times 1) \\ \hline 4.128 \end{array}$ 5.NOS.B.7d: Apply understanding of multiplication and division as inverse operations to think of a division problem as multiplication problem with a missing factor. For example, when solving $1.28 \div 0.4$, a student can think of $? \times 0.4 = 1.28$. The student can use known facts and understanding of place value patterns to find the missing factor, 3×0.4 and $0.2 \times 0.4 = 0.08$ so $1.28 \div 0.4 = 3.2$. 5.NOS.B.7e: Students can explain how their representation or strategy accurately represents multiplication. For example, if using the area model to multiply 1.3×1.2, a student may explain that each factor was decomposed into ones and tenths and then used the distributive property to multiply each to get total product.	

Previously 5.NBT.B.7

5.NOS.C USE EQUIVALENCE FRACTIONS AS A STRATEGY TO ADD AND SUBTRACT FRACTIONS.**5.NOS.C.8: Add and subtract fractions with unlike denominators.**

- a. Estimate sums and differences by reasoning about benchmark numbers and assess reasonableness of answers (e.g., $\frac{2}{3} + \frac{3}{4}$ will be more than 1 because both fractions are greater than $\frac{1}{2}$).
- b. Apply and extend whole number addition and subtraction strategies (e.g. counting on, making a whole, partial sums, compensation, properties of operations, the inverse relationship between addition and subtraction) to add and subtract fractions.
- c. Use equivalent fractions to produce an equivalent sum or difference of fractions with like denominators (e.g., $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$). In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Add and subtract fractions with like denominators (4.NOS.E.12). - Understand and generate equivalent fractions (4.NOS.D.9). Where are students going? - Solve problems in context by writing and solving equations with nonnegative rationale numbers and interpreting the solutions in context (6.AT.C.8).	- Grade 5 students will work with fractions less than 1, equal to 1, or greater than 1 (including mixed numbers). - Use “fractions greater than 1.” Avoid using “improper fractions” as this phrase can be confusing/misleading. - Continue to emphasize the importance of the size of the whole. Students must recognize that when joining or separating parts, all parts must refer to the same whole. - Students develop an understanding of addition and subtraction of fractions with unlike denominators through a balance of concrete, semi-concrete, and abstract representations. Avoid moving to abstract approaches too quickly without ensuring students have sufficient experience with concrete and semi-concrete approaches. 5.NOS.C.8b: Use partial sums to add fractions. For example, a student may use partial sums to find $\begin{array}{r} \frac{11}{12} + 2\frac{1}{2} \\ \swarrow \quad \searrow \\ 1\frac{11}{12} \quad 2\frac{1}{2} = \frac{6}{2} \\ \hline 1 + 2 = 3 \quad \frac{11}{12} + \frac{6}{12} = \frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12} \\ \hline 3 + 1\frac{5}{12} = 4\frac{5}{12} \end{array}$ <i>Continued on next page</i>	- Make explicit connections between different approaches. - Use strategies flexibly, making decisions on which strategy to use based on the numbers or context of the given problem (SMP 5). - Engage in Number Talks to promote flexible thinking and provide opportunities to explain their thinking and make connections to other strategies. - Use visual, symbolic, or contextual representations to explain how to add or subtract fractions with unlike denominators. - Apply estimation strategies to assess the reasonableness of sums and differences. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Number and Operation Sense milestones in the Fractions & Decimals strand of the Numeracy Development Framework.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	5.NOS.C.8b: Apply inverse relationship between addition and subtraction to add and subtract fractions. For example, a student may think of $\frac{2}{3} + ? = \frac{8}{2}$ to solve $\frac{8}{2} - \frac{2}{3}$. 5.NOS.C.8c: Apply understanding of equivalent fractions to write equivalent expressions with like denominators to add or subtract. Model the use of precise equations. $\frac{7}{8} + \frac{3}{4}$ $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$ $\frac{7}{8} + \frac{6}{8} = \frac{13}{8}$ $\frac{13}{8} = 1\frac{5}{8}$ - Intentionally select fractions that encourage the use of specific strategies. This supports conceptual understanding and promotes efficient approaches rather than rote calculation. Avoid fractions that distract from mathematical focus by making computation unnecessarily cumbersome. - Students are not required to simplify fractions, however, they should be able to recognize equivalent fractions when necessary. - Emphasize that fractions are numbers. Avoid too much discussion of “the top and the bottom” (numerator and denominator) as this implies that these numbers have separate values which may lead to inappropriate whole number reasoning.	

Previously 5.NF.A.1

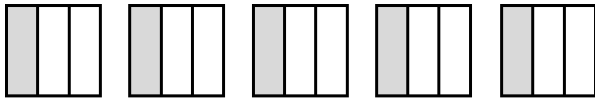
5.NOS.C.9: Solve problems in context involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators using visual fraction models and/or equations to represent the problem.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Add and subtract fractions with like denominators (4.NOS.E.12). - Understand and generate equivalent fractions (4.NOS.D.9). Where are students going? - Solve problems in context by writing and solving equations with nonnegative rationale numbers and interpreting the solutions in context (6.AT.C.8).	- Grade 5 students will work with fractions less than 1, equal to 1, or greater than 1 (including mixed numbers). - Use “fractions greater than 1.” Avoid using “improper fractions” as this phrase can be confusing/misleading. - Continue to emphasize the importance of the size of the whole. Students must recognize that when joining or separating parts, all parts must refer to the same whole. - Apply a variety of strategies (5.NOS.C.8) to solve problems in context involving addition and subtraction of fractions. - Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students’ everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved. Refer to Addition and Subtraction Situations for examples of problem types. Grade 4 students work with unknowns in all positions. - Students are not required to simplify fractions, however, they should be able to recognize equivalent fractions when necessary.	- Interpret a variety of contexts and describe what is happening in their own words (SMP 1). - Discuss the meaning of quantities in context (SMP 2). - Determine the information needed to answer the mathematical question. - Represent contexts by selecting and applying appropriate representations (e.g., visual fraction models or equations). Students should develop flexibility in their use of various representations. - Apply estimation strategies to assess the reasonableness of sums and differences. - Apply understanding of adding and subtracting fractions with unlike denominators to interpret data presented in line plots (5.DS.A.1).

Previously 5.NF.A.2

5.NOS.D APPLY AND EXTEND PREVIOUS UNDERSTANDING OF MULTIPLICATION AND DIVISION TO MULTIPLY AND DIVIDE FRACTIONS.

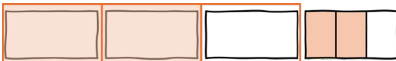
5.NOS.D.10: Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$) in context using visual fraction models and equations to represent the problem (e.g., interpret $\frac{3}{4}$ as the result of dividing 3 by 4 so when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Interpret multiplication situations as a comparison and solve multiplicative comparison problems in context (4.A.T.A.1; 4.A.T.A.2). Where are students going? - Represent and use unit rates, written as fractions with whole number numerators and denominators, in context (6.A.T.A.2).	- Grade 5 students will work with fractions less than 1, equal to 1, or greater than 1 (including mixed numbers). - Use “fractions greater than 1.” Avoid using “improper fractions” as this phrase can be confusing/misleading. - Use authentic contexts to support students in building conceptual understanding. Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students’ everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved. - Use visual models to represent problems in context. For example, a student may draw a picture to represent the following problem: A relay team will run a total of 5 miles. There are 3 people on the team that will each run an equal amount. How many miles will each person run?  $5 \div 3 = \frac{5}{3} = 1\frac{2}{3} \text{ miles}$ - Students are not required to simplify fractions, however, they should be able to recognize equivalent fractions when necessary.	- Interpret a variety of contexts and describe what is happening in their own words (SMP 1). - Discuss the meaning of quantities in context (SMP 2). - Determine the information needed to answer the mathematical question. - Represent contexts by selecting and applying appropriate representations (e.g., visual fraction models or equations). Students should develop flexibility in their use of various representations. - Make explicit connections between representation and context. Students should also be encouraged to make connections between different approaches for solving the same problem. - Apply estimation strategies to assess the reasonableness.

Previously 5.NF.B.3

5.NOS.D.11: Apply and extend previous understandings of multiplication to multiply a whole number by a fraction ($\frac{c}{d} \times b$).

- a. Estimate products by reasoning about benchmark numbers and assess reasonableness of answers (e.g., $\frac{3}{4} \times 2$ will be less than 2 because I am creating a fractional part of 2).
- b. Multiply a whole number by a fraction (e.g., interpret $\frac{c}{d} \times b$ as $\frac{c}{d}$ of b when $\frac{c}{d}$ is greater than or less than 1).

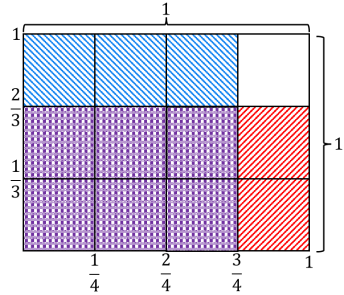
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Multiply whole numbers (4.NOS.B.5). - Multiply a fraction by a whole number, $b \times \frac{c}{d}$ (4.NOS.E.13). Where are students going? - Multiply a fraction by a fraction (5.NOS.D.12). - Divide fractions by fractions (6.NOS.A.1). - Solve problems in context by writing and solving equations and interpreting the solutions in context (6.AT.C.8).	- Grade 5 students will work with fractions less than 1 or greater than 1 (including mixed numbers). - Use “fractions greater than 1.” Avoid using “improper fractions” as this phrase can be confusing/misleading. - Pose a variety of examples to support students in recognizing patterns. Encourage students to describe what happens to the product when multiplying fractions greater than and less than 1. - Use concrete or visual models to help students visualize a fractional amount of a whole number. For example, the visual model below represents $\frac{2}{3} \times 4$. The model shows 4 wholes, each partitioned into thirds. Two-thirds of each whole is shaded, for a total of $\frac{8}{3}$.  Note, that this model could also be used to represent $4 \times \frac{2}{3}$. Discuss with students how the same model can be used to represent two different expressions. - The expression, $\frac{2}{3} \times 4$, can also be represented with the visual model below. In this model, $\frac{2}{3}$ of 3 is shown with the larger orange rectangle. Then $\frac{2}{3}$ of 1 is shown with the final rectangle.  Discuss the similarities and differences between this model and the previous model above. <i>Continued on next page</i>	- Make explicit connections between different approaches. - Use strategies flexibly, making decisions on which strategy to use based on the numbers or context of the given problem (SMP 5). - Engage in Number Talks to promote flexible thinking and provide opportunities to explain their thinking and make connections to other strategies. - Use visual, symbolic, or contextual representations to explain how to multiply a whole number by a fraction. - Apply estimation strategies to assess the reasonableness of products. - Apply understanding of multiplication of a whole number by a fraction to solve problems in context (5.NOS.D.15). Engage in authentic contexts that involve saving or budgeting (e.g., Each month the worker saves $\frac{1}{4}$ of his monthly pay. Is he earns \$1,750 in March, how much will he save in March?). \$ Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Number and Operation Sense milestones in the Fractions & Decimals strand of the Numeracy Development Framework.

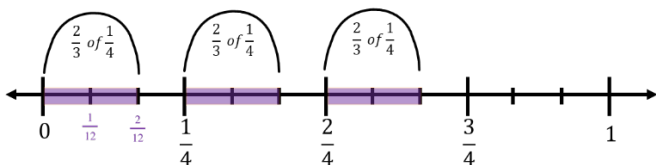
Content Progression	Examples and Clarifications	Students should have the opportunity to...
	- A bar model can also be used to represent multiplication of a whole number by a fraction. For example, the bar model below represents $\frac{2}{3} \times 18$. The whole is 18. The whole (represented by bar) is partitioned into 3 equal parts. Each third is equivalent to 6 students, so two-thirds of the 18 students is 12 students. 18 students 6 6 6 12 students - The Distributive Property of Multiplication can be applied to multiply a whole number by a fraction that is a mixed number. For example, a student may use the Distribute Property of Multiplication to multiply $2\frac{1}{3} \times 6$. $\begin{array}{r} 2\frac{1}{3} \times 6 \\ \swarrow \searrow \\ 2 + \frac{1}{3} \\ (2 \times 6) + (\frac{1}{3} \times 6) \\ 12 \quad + \quad 2 \\ \hline 14 \end{array}$ - Students are not required to simplify fractions, however, they should be able to recognize equivalent fractions when necessary.	

Previously 5.NF.B.4

5.NOS.D.12: Apply and extend previous understandings of multiplication to multiply a fraction by a fraction.

- a. Estimate products by reasoning about benchmark numbers and assess reasonableness of answers (e.g., $\frac{1}{2} \times \frac{3}{4}$ will be less than $\frac{3}{4}$ because I will have one half of $\frac{3}{4}$).
- b. Apply and extend whole number multiplication strategies (e.g. area models).
- c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths.

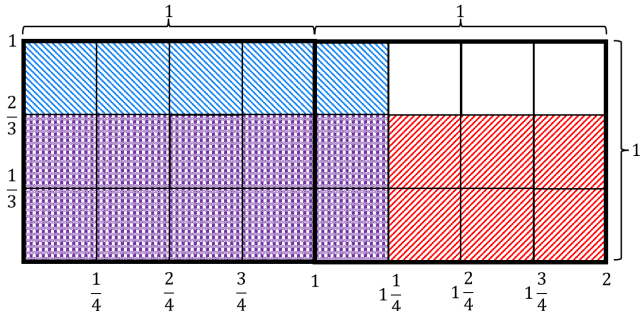
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Multiply whole numbers (4.NOS.B.5). - Multiply a fraction by a whole number, $b \times \frac{c}{d}$ (4.NOS.E.13). - Multiply a whole number by a fraction (5.NOS.D.11). Where are students going? - Solve problems in context involving multiplication of fractions and mixed numbers, including multiplication as scaling (5.NOS.D.14; 5.NOS.D.15). <i>Continued on next page</i>	- This standard focuses on multiplying fractions less than 1 to support students' foundational understanding. 5.NOS.D.13 focuses on extending understanding to multiply fractions greater than 1. - Pose a variety of examples to support students in recognizing patterns. Encourage students to describe what happens to the product when multiplying two fractions less than 1. Avoid directing students to multiply numerators and denominators without student exploration and discovery. - Use concrete models, such as square unit tiles to represent and multiply two fractions. For example, tiles can be used to represent and multiply $\frac{2}{3} \times \frac{3}{4}$. One dimension is partitioned into thirds (represented by 3 tiles), and the other dimension is partitioned into fourths (represented by 4 tiles). Each tile represents $\frac{1}{12}$ of the whole. Therefore, $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$.  <i>Continued on next page</i>	- Use visual, symbolic, or contextual representations to explain how to multiply a fraction by a fraction. - Apply estimation strategies to assess the reasonableness of products. - Analyze a variety of problems involving the multiplication of two fractions. Students should be encouraged to identify regularities or repeated calculations to make generalization about how operations, specifically multiplication of fractions, behave (SMP 8). - Justify why the shortcut or procedure ($\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$) for multiplying fractions works. - Apply understanding of multiplication of a fraction by a fraction to solve problems in context, such as area, sharing, or measurement (5.NOS.D.15). Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Number and Operation Sense milestones in the Fractions & Decimals strand of the Numeracy Development Framework.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Solve problems in context by writing and solving equations and interpreting the solutions in context (6.A.T.C.8).	- Number lines can also be used to represent and solve $\frac{2}{3} \times \frac{3}{4}$. If $\frac{2}{3}$ of $\frac{1}{4}$ is $\frac{2}{12}$ then $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{6}{12}$.  - As students notice patterns, encourage students to describe the patterns using mathematical language. Students may derive and test the procedure for multiplying two fractions ($\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$) based on the patterns they notice. Avoid introducing the procedure before student exploration and discovery. - Students are not required to simplify fractions, however, they should be able to recognize equivalent fractions when necessary.	

Previously 5.NF.B.4

5.NOS.D.13: Apply and extend previous understandings of multiplication to multiply a fraction by a fraction, with one or both factors greater than 1.

- a. Estimate products by reasoning about benchmark numbers and assess reasonableness of answers.
- b. Apply and extend whole number multiplication strategies (e.g. area model, partial products).

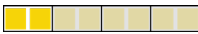
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Multiply whole numbers (4.NOS.B.5). - Multiply a fraction by a fraction (5.NOS.D.12). Where are students going? - Solve problems in context involving multiplication of fractions and mixed numbers, including multiplication as scaling (5.NOS.D.14; 5.NOS.D.15). - Solve problems in context by writing and solving equations and interpreting the solutions in context (6.AT.C.8).	- This standard focuses on multiplying two fractions, with one or both factors greater than 1 (including mixed numbers). Use “fractions greater than 1.” Avoid using “improper fractions” as this phrase can be confusing/misleading. - Pose a variety of examples to support students in recognizing patterns. Encourage students to describe what happens to the product when multiplying by a fraction greater than 1. Avoid directing students to multiply numerators and denominators without student exploration and discovery. 5.NOS.D.13b: Apply students’ previous understanding of area models to multiply fractions greater than 1. For example, an area model can be used to represent and multiply $\frac{2}{3} \times 1\frac{1}{4}$. Since one factor is greater than 1, the area model shows 2 wholes. The wholes are then partitioned horizontally to represent thirds (purple and red shading), and the vertically to represent fourths (blue and purple shading). Each square represents $\frac{1}{12}$ of the whole. Therefore, $\frac{2}{3} \times 1\frac{1}{4} = \frac{10}{12}$.  <i>Continued on next page</i>	- Explore how models used to multiply a fraction by a fraction (both less than 1) can be applied to multiply a fraction by a fraction with one or both factors greater than 1. - Use visual, symbolic, or contextual representations to explain how to multiply a fraction by a fraction, when one or both factors greater than 1. - Apply estimation strategies to assess the reasonableness of products. - Apply understanding of multiplication of a fraction by a fraction (with one or both factors greater than 1) to solve problems in context (5.NOS.D.15).

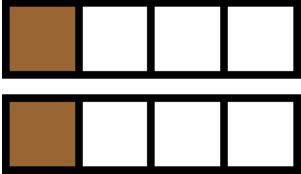
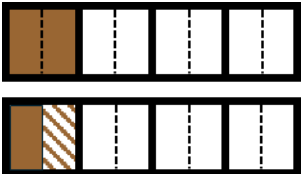
Content Progression	Examples and Clarifications	Students should have the opportunity to...
	When using area models with fractions greater than 1, it is important that students pay attention to the whole. In the example above, each whole represented by the dark border. Each whole is partitioned into 12 equal pieces representing twelfths. A student who misinterprets the longer rectangle as the whole, will inaccurately identify the product. • Apply students' previous understanding of partial products to multiply fractions greater than 1. When working with fractions greater than 1, students will decompose the fractions into parts, often whole numbers and fractional part. For example, when multiplying $1\frac{1}{3} \times 2\frac{3}{8}$, a student may begin by decomposing each factor, $1\frac{1}{3} = 1 + \frac{1}{3}$ and $2\frac{3}{8} = 2 + \frac{3}{8}$, and then follow the steps below to solve. $1 \times 2 = 2$ $1 \times \frac{3}{8} = \frac{3}{8}$ $\frac{3}{8} = \frac{9}{24}$ $\frac{1}{3} \times 2 = \frac{2}{3}$ $\frac{2}{3} = \frac{16}{24}$ $\frac{1}{3} \times \frac{3}{8} = \frac{3}{24}$ $2 + \frac{9}{24} + \frac{16}{24} + \frac{3}{24} = 2 + \frac{28}{24} = 2 + \frac{24}{24} + \frac{4}{24} = 3\frac{4}{24}$ • As students notice patterns, encourage students to describe the patterns using mathematical language. Students may derive and test the procedure for multiplying two fractions ($\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$) based on the patterns they notice. Avoid introducing the procedure before student exploration and discovery. • Students are not required to simplify fractions, however, they should be able to recognize equivalent fractions when necessary.	

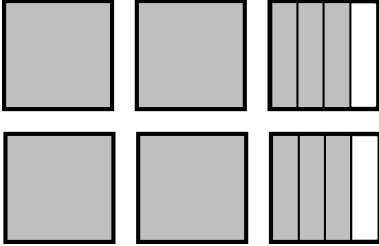
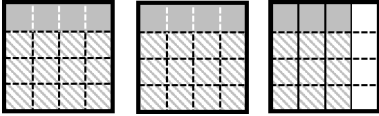
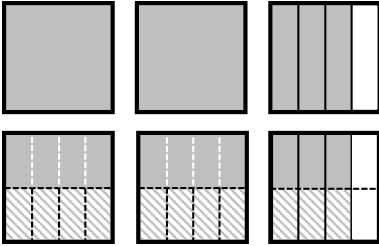
Previously 5.NF.B.4

5.NOS.D.14: Interpret multiplication as scaling (sizing) in context.

- Compare the size of a product to the size of one factor based on the size of the other factor, without performing the indicated multiplication.
- Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relate the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the Identity Property of Multiplication.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Interpret a multiplication situation as a comparison and solve multiplicative comparison problems in context (4.A.T.A.1; 4.A.T.A.2). - Multiply a fraction by a whole number, $b \times \frac{c}{d}$ (4.NOS.E.13). Where are students going? - Use ratio language in context to describe a ratio relationship between two quantities (6.A.T.A.1)	- This standard focuses on reasoning about the size of products based on the size of the factors, including whole numbers and fractions less than, equal to and greater than 1 (including mixed numbers). - Use “fractions greater than 1.” Avoid using “improper fractions” as this phrase can be confusing/misleading - Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students’ everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved. - Pair contexts with visual models to develop students conceptual understanding and support sense making. A cookie recipe requires 8 tablespoons of butter. How many tablespoons of butter are needed for 2 batches?  $2 \times 8 = 16 \text{ tablespoons of butter}$ A cookie recipe requires 8 tablespoons of butter. How many tablespoons of butter are needed for a quarter of a batch?  $\frac{1}{4} \times 8 = 2 \text{ tablespoons of butter}$ A cookie recipe requires 8 tablespoons of butter. How many tablespoons of butter are needed for one and a half batches?  $1\frac{1}{2} \times 8 = 12 \text{ tablespoons of butter}$ <i>Continued on next page</i>	- Predict the size of the product and explain reasoning without calculation. - Use visual, symbolic, or contextual representations to explain how the size of the factors impacts the size of the product. - Use identified patterns to form general rules and justify their validity using mathematical reasoning. - Apply understanding of multiplication as scaling to solve problems in context (5.NOS.D.15).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	A cookie recipe requires $\frac{1}{4}$ cup of brown sugar. How many cups of brown sugar are needed for 2 batches?  $2 \times \frac{1}{4} = \frac{2}{4} \text{ cup of brown sugar}$ A cookie recipe requires $\frac{1}{4}$ cup of brown sugar. How many cups of brown sugar are needed for a quarter of a batch?  $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16} \text{ cup of brown sugar}$ A cookie recipe requires $\frac{1}{4}$ cup of brown sugar. How many cups of brown sugar are needed for $1\frac{1}{2}$ batches?  $1\frac{1}{2} \times \frac{1}{4} = \frac{3}{8} \text{ cup of brown sugar}$ <i>Continued on next page</i>	

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	A cookie recipe requires $2\frac{3}{4}$ cups of flour. How many cups of flour are needed for 2 batches?  $2 \times 2\frac{3}{4} = 4 + \frac{6}{4} = 5\frac{2}{4} \text{ cups of flour}$ A cookie recipe requires $2\frac{3}{4}$ cups of flour. How many cups of flour are needed for a quarter of a batch?  $\frac{1}{4} \times 2\frac{3}{4} = \frac{8}{16} + \frac{3}{16} = \frac{11}{16} \text{ cups of flour}$ A cookie recipe requires $2\frac{3}{4}$ cups of flour. How many cups of flour are needed for $1\frac{1}{2}$ batches?  $1\frac{1}{2} \times 2\frac{3}{4} = 2 + \frac{3}{4} + 1 + \frac{3}{8} = 4\frac{1}{8} \text{ cups of flour}$	

Previously 5.NF.B.5

5.NOS.D.15: Solve problems in contexts involving multiplication of fractions and mixed numbers, (e.g., by using visual fraction models or equations to represent the problem).

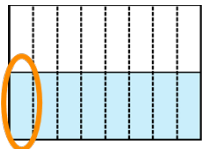
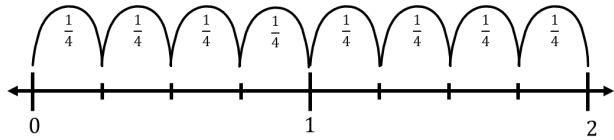
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Multiply a fraction by a whole number, $b \times \frac{c}{d}$ (4.NOS.E.13). - Multiply a whole number by a fraction and fraction by fraction including fractions less than, equal to, or greater than 1 (5.NOS.D.11; 5.NOS.D.12; 5.NOS.D.13). Where are students going? - Solve problems in context by writing and solving equations and interpreting the solutions in context (6.AT.C.8).	- This standard involves multiplying fractions less than, equal to, or greater than 1 (including mixed numbers). - Use “fractions greater than 1.” Avoid using “improper fractions” as this phrase can be confusing/misleading. - Students represent and solve problems in context using a balance of concrete, semi-concrete, and abstract approaches. Model various approaches and encourage flexibility. Avoid modeling or requiring a single approach. - Present authentic contexts to support students understanding. Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students’ everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved. The robotics team builds a test ramp that is $4\frac{1}{3}$ feet long. Their robot completes $\frac{3}{4}$ of the ramp before running out of battery. How far did the robot travel?	- Make sense of and interpret problems in context using numbers, diagrams, and equations and identify what each quantity represents (SMP 1; SMP 2). - Represent a variety of contexts mathematically by selecting and applying appropriate concepts, tools, and representations to define variables, model relationships, and/or identify constraints. - Use strategies flexibly, making decisions on which strategy or method to use based on the numbers or context of the given problem (SMP 5). Make explicit connections between different approaches. - Apply understanding of multiplication as scaling (sizing) and the impact that size of factors has on size of product to estimate products and assess the reasonableness of products in context.

Previously 5.NF.B.6

5.NOS.D.16: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions in context.

- Apply estimation strategies to estimate quotients and assess reasonableness of answers.**
- Use visual fraction models and equations to interpret division of a unit fraction by a non-zero whole number and compute quotients.**
- Use visual fraction models and equations to interpret division of a whole number by a unit fraction and compute quotients.**
- Represent and explain the computation by connecting visual fraction models and/or equations to the meaning of division with unit fractions.**

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Multiply a fraction by a whole number, $b \times \frac{c}{d}$ (4.NOS.E.13). Where are students going? - Divide fractions by fractions in context (6.NOS.A.1). - Solve problems in context by writing and solving equations and interpreting the solutions in context (6.AT.C.8).	- This standard focuses only on division situations involving a <i>unit fraction</i> (a fraction with a numerator of 1) and a <i>whole number</i>. Divide unit fraction by whole number: $\frac{1}{b} \div c$ Divide whole number by unit fraction: $c \div \frac{1}{b}$ - Division of fractions is a new application for Grade 5 students. In Grade 4, students applied understanding of operations to divide whole numbers and to multiply a fraction by a whole number. Students will apply this understanding to dividing units fractions and whole numbers. - Identify fractions that encourage the use of specific strategies. This supports conceptual understanding and promotes efficient approaches rather than rote calculation. Avoid fractions that distract from mathematical focus by making computation unnecessarily cumbersome. - Use authentic contexts to support students in building conceptual understanding of dividing fractions by whole numbers. Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students' everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved. <i>Continued on next page</i>	- Explore how visual models and equations that are used to divide whole numbers and/or multiply whole numbers and fractions can be applied to divide unit fractions by whole numbers and whole numbers by unit fractions. - Make explicit connections between different approaches. - Use strategies flexibly, making decisions on which strategy to use based on the numbers or context of the given problem (SMP 5). - Use visual, symbolic, or contextual representations to explain why a visual model or equation accurately represents a given division problem. - Apply estimation strategies to assess the reasonableness of sums and differences.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	5.NOS.D.16b: A student may use the area model and equation below to represent and solve the following problem. The basketball coach brings $\frac{1}{2}$ gallon of water to fill water bottles before practice. If the coach fills 8 water bottles, how many gallons will be poured into each bottle?  $\frac{1}{2} \div 8 = \frac{1}{16} \text{ gallon of water in each bottle}$ 5.NOS.D.16c: A student may use the number line and equation below to represent and solve the following problem. The student has a goal of reading 2 hours each week. If the student reads one-quarter hour each week, will the student meet their goal?  $2 \div \frac{1}{4} = 8$ <i>The student will meet their goal after 8 days, not a week.</i> 5.NOS.D.16d: Students can explain how their representation or strategy accurately represents division. For example, if using a number line to represent and solve $2 \div \frac{1}{4} = 8$, a student may explain that $2 \div \frac{1}{4} = 8$ can be thought of as $g \times \frac{1}{4} = 2$. The number line shows the number of groups of $\frac{1}{4}$ that are needed to equal 2 wholes.	

Previously 5.NF.B.7

Algebraic Thinking (AT)

Algebraic Thinking (AT) Clusters	Connections to the Numeracy Development Framework
5.AT.A Write and interpret numerical expressions.	Standards within this cluster directly contribute* to the Grade 5 Algebraic Thinking milestones within the Creating Equations & Inequalities strand of the Numeracy Development Framework.
5.AT.B Analyze patterns and relationships.	Standards within this cluster directly contribute* to the Grade 5 Algebraic Thinking milestones within the Functional Reasoning strand of the Numeracy Development Framework.

***Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

****Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.

5.AT.A WRITE AND INTERPRET NUMERICAL EXPRESSIONS.**5.AT.A.1: Interpret and evaluate numerical expressions with grouping symbols (e.g. parentheses, brackets, or braces).**

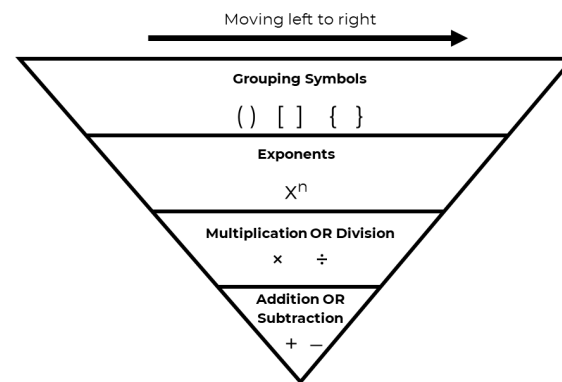
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - This is the first formal introduction of grouping symbols. Where are students going? - Write expressions that record calculations with numbers and interpret numerical expressions without evaluating them (5.AT.A.2). - Write numerical expressions involving whole number exponents and positive rational number bases (6.AT.B.5). - Write, read, and evaluate expressions in which letters stand for numbers (6.AT.B.6).	- Numerical expressions must involve calculations that are in alignment with Grade 5 NOS expectations, including whole numbers, decimals, and fractions. - Present a variety of contexts that model the order of operations. Avoid giving students the order of operations or relying on pneumatic devices (e.g., PEMDAS) without student exploration and discovery. A school group visits a museum. They will purchase museum tickets that cost \$11 each and lunches that cost \$7.50 for each person. If there are 24 people in the school group, how much money is spent at the museum? $24 \times (11 + 7.5)$ A school group visits a museum. They will purchase museum tickets that cost \$11 each and lunches that cost \$7.50 for each person. The group will receive a group discount of \$50. If there are 24 people in the school group, how much money is spent at the museum? $24 \times (11 + 7.5) - 50$ A school group visits a museum. They will purchase museum tickets that cost \$11 each. They will purchase pizzas for lunch that cost a total of \$140. If there are 24 people in the school group, how much money is spent at the museum? $24 \times 11 + 140$ - Make connections to Associative Property of Addition (addends can be grouped in any way without changing the sum), the Associative Property of Multiplication (factors can be grouped in any way without changing the product), and the Distributive Property of Multiplication (multiplying a number by a sum is the same as multiplying the number by each addend separately and adding the products). <i>Continued on next page</i>	- Engage with varying problems in context to explore order of operations. Through exploration, students begin to use the structure of grouping symbols to accurately represent operations (SMP 7). - Examine scenarios to identify location of grouping symbols or match expressions to scenarios. - Compare expressions with and without grouping symbols to explain the difference(s). - Apply their understanding of numerical expressions with grouping symbols to interpret and evaluate expressions related to geometric formulas for perimeter, area, and volume. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Algebraic Thinking milestones in the Creating Equations & Inequalities strand of the Numeracy Development Framework.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	Use visuals to represent the sequence in which mathematical operations should be performed. Emphasize that the order of operations is applied from left to right. Avoid relying on mnemonic devices (e.g., PEMDAS) without student exploration and reasoning, as this can lead to the misconception that multiplication always comes before division, or addition before subtraction, rather than understanding that these operations are completed in the order they appear in the expression. Moving left to right Grouping Symbols () [] { } Exponents x^n Multiplication OR Division $\times \div$ Addition OR Subtraction $+ -$	

Previously 5.OA.A.1

5.AT.A.2: Write expressions that record calculations with numbers and interpret numerical expressions without evaluating them.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Interpret and evaluate numerical expressions with grouping symbols (5.AT.A.1). Where are students going? - Write numerical expressions involving whole number exponents and positive rational number bases (6.AT.B.5). - Write, read, and evaluate expressions in which letters stand for numbers (6.AT.B.6).	- Numerical expressions must involve calculations that are in alignment with Grade 5 NOS expectations, including whole numbers, decimals, and fractions. - Students will write numerical expressions to describe mathematical calculations. For example, a student writes $2 \times (8 + 7)$ to express the calculation “add 8 and 7, then multiply by 2.” - Students will describe the actions represented by a numerical expression. For example, a student explains that $3 \times (18932 + 921)$ means “three times as large as the sum of 18932 and 921. - Make connections to Associative Property of Addition (addends can be grouped in any way without changing the sum), the Associative Property of Multiplication (factors can be grouped in any way without changing the product), and the Distributive Property of Multiplication (multiplying a number by a sum is the same as multiplying the number by each addend separately and adding the products). - Use visuals to represent the sequence in which mathematical operations should be performed. Emphasize that the order of operations is applied from left to right. Avoid relying on mnemonic devices (e.g., PEMDAS) without student exploration and reasoning, as this can lead to the misconception that multiplication always comes before division, or addition before subtraction, rather than understanding that these operations are completed in the order they appear in the expression (see diagram to the right).	- Write numerical expressions given varying mathematical calculations either expressed verbally or written in words. - Interpret a variety of expressions to explain what they mean in words and to identify what operation happens first. - Compare expressions with and without grouping symbols to explain the difference(s). - Use expressions to model an authentic context using mathematics (SMP 4). - Engage in problem contexts that involve spending, saving, and earning money (e.g., write an expression to represent the situation: Your friend earns \$12 for pet sitting and \$60 for babysitting each week. After 5 weeks, she will put half of her earnings into her savings account and the other half will be used to purchase a new bike). \$ Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Algebraic Thinking milestones in the Creating Equations & Inequalities strand of the Numeracy Development Framework.

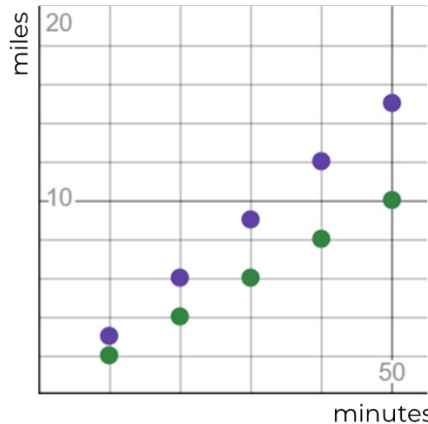


Previously 5.OA.A.2

5.AT.B ANALYZE PATTERNS AND RELATIONSHIPS.**5.AT.B.3: Generate and analyze number patterns.**

- Generate a number pattern that follows a given rule and identify relationships between the numbers within the pattern.
- Generate two numerical patterns using two given rules and identify relationships between corresponding terms.
- Use tables, ordered pairs, and graphs to represent the relationship between quantities.

Content Progression	Examples and Clarifications	Students should have the opportunity to...																		
Where are students coming from? - Identify and explain arithmetic patterns (3.AT.A.3). Where are students going? - Use ratio language to describe a ratio relationship between two quantities (6.AT.A.1). - Write, read, and evaluate expressions in which letters stand for numbers (6.AT.B.6). - Use variables to represent two quantities that change in relationship to one another (6.AT.D.11).	- Number patterns must involve calculations that are in alignment with Grade 5 NOS expectations, including whole numbers, decimals, and fractions. 5.AT.B.3a: As students begin working with number patterns, they may need to generate multiple terms in a pattern to be able to answer questions as shown in the example. When parking in the parking garage, \$6 is added to the cost for every hour parked in the garage. How much will it cost to be parked in the garage for 8 hours?<table><tr><td># of hours</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>cost</td><td>\$6</td><td>\$12</td><td>\$18</td><td>\$24</td><td>\$30</td><td>\$36</td><td>\$42</td><td>\$48</td></tr></table> Encourage students to identify relationships between numbers in the patterns to more efficiently answer questions. When answering the question above, a student may recognize that in addition to \$6 being added for each hour, there is also a relationship between the number of hours and the cost. The cost is always six times as much as the number of hours. Therefore, the number of hours can be multiplied by 6 to find the cost (e.g., $8 \times 6 = 48$). 5.AT.B.3b: Corresponding terms refer to the terms in the same position in each pattern. Continued on next page	# of hours	1	2	3	4	5	6	7	8	cost	\$6	\$12	\$18	\$24	\$30	\$36	\$42	\$48	- Use tables and graphs to represent and explore relationships between quantities and corresponding terms in patterns. - Predict terms before calculating. - Describe and compare patterns. - Create number pattern(s) to model authentic context(s) using mathematics (SMP 4). - Apply understanding of number patterns to graph and name coordinate points (5.GR.D.6) and to represent problems and interpret coordinate values in context (5.GR.D.7). - Engage in problem contexts that involve spending, saving, and earning money (e.g., You save \$8 each week. Your friend saves half as much each week. How do your savings compare to your friend’s savings). \$ Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Algebraic Thinking milestones in the Functional Reasoning strand of the Numeracy Development Framework.
# of hours	1	2	3	4	5	6	7	8												
cost	\$6	\$12	\$18	\$24	\$30	\$36	\$42	\$48												

Content Progression	Examples and Clarifications	Students should have the opportunity to...																		
	5.AT.B.3b: As students transition to working with two rules to generate number patterns, they can consider way to organize the patterns to help make relationships more apparent, as shown in the example below. Two friends ride their bikes for exercise. Friend A rides 2 miles every 10 minutes. Friend B rides 3 miles every 10 minutes. <table border="1" data-bbox="653 496 1033 745"> <thead> <tr> <th># of minutes</th><th>Friend A # of miles</th><th>Friend B # of miles</th></tr> </thead> <tbody> <tr> <td>10</td><td>2</td><td>3</td></tr> <tr> <td>20</td><td>4</td><td>6</td></tr> <tr> <td>30</td><td>6</td><td>9</td></tr> <tr> <td>40</td><td>8</td><td>12</td></tr> <tr> <td>50</td><td>10</td><td>15</td></tr> </tbody> </table> The table supports students in identifying the relationship between corresponding terms. For example, Friend B rides 1.5 times as far as Friend A because $4 \times 1.5 = 6$ or $10 \times 1.5 = 15$. 5.AT.B.3c: When working with ordered pairs and graphs, Grade 5 students will work within Quadrant I of the coordinate plane, focusing on positive numbers only. 	# of minutes	Friend A # of miles	Friend B # of miles	10	2	3	20	4	6	30	6	9	40	8	12	50	10	15	
# of minutes	Friend A # of miles	Friend B # of miles																		
10	2	3																		
20	4	6																		
30	6	9																		
40	8	12																		
50	10	15																		

Previously 5.OA.B.3

Geometric Reasoning and Measurement (GR)

Geometric Reasoning Clusters	Connections to the Numeracy Development Framework
5.GR.A Understand relationships between measurement units.	Standards within this cluster directly contribute* to the Grade 5 Geometric Reasoning and Measurement milestones within the Measurement strand of the Numeracy Development Framework.
5.GR.B Classify two-dimensional figures based on properties.	Standards within this cluster directly contribute* to the Grade 5 Geometric Reasoning and Measurement milestones within the Classifying Figures strand of the Numeracy Development Framework.
5.GR.C Understand concepts of volume and relate volume to multiplication	Standards within this cluster directly contribute* to the Grade 5 Geometric Reasoning and Measurement milestones within the Reasoning with Geometric Structure strand of the Numeracy Development Framework.
5.GR.D Graph points on a coordinate plane to solve contextual and mathematical problems.	Standards within this cluster directly contribute* to the Grade 5 Geometric Reasoning and Measurement milestones within the Spatial Reasoning strand of the Numeracy Development Framework.

***Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

****Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.

5.GR.A UNDERSTAND RELATIONSHIPS BETWEEN MEASUREMENT UNITS.

5.GR.A.1: Apply the relationship between measurement units within a given measurement system to convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions in solving multi-step problems.

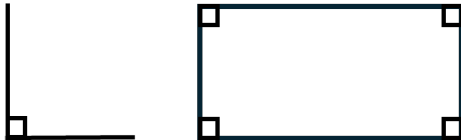
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Convert measurements from a larger unit to smaller unit and use this to solve problems involving the four operations (4.GR.A.1; 4.GR.A.2). - Add, subtract, multiply, divide decimals to hundredths (5.NOS.B.6; 5.NOS.B.7). Where are students going? - Use ratio and rate reasoning to solve problems in context (6.A.T.A.3).	- Conversions and multi-step problems must involve calculations that are in alignment with Grade 5 NOS expectations, including whole numbers, decimals, and fractions. - Grade 5 students convert measurements within the same measurement system (e.g., feet to inches, pounds to ounces). Students are not expected to convert across measurement systems (e.g., feet to centimeters). - When working in the metric measurement system, use decimals instead of fractions. When working in the U.S. customary measurement system, use fractions instead of decimals. - Number lines and tables can also be used to convert measurements and record equivalences. - Present multi-step problems in authentic contexts. Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students' everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved. The Nature Club will use wood boards to create a garden. The maximum area allowed for the garden is 108 square feet. The local hardware store donated 4 boards measuring 2 yards, 3 yards, 3 yards, and 6 yards. What is the largest rectangular garden that can be made with the boards provided? <i>Continued on next page</i>	- Use unit labels appropriately to accurately communicate relationships between measurements units (SMP 6). - Make connections to place value understanding and relationship between place values (5.NOS.A.1) to describe the relationship between metric units. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Geometric Reasoning and Measurement milestones in the Measurement strand of the Numeracy Development Framework.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	Focus on the relationship between units (e.g., e.g., “a meter is 100 times as long as a centimeter,” “a centimeter is one hundredth of a meter”). Use precise language such as “multiply by 100” or “divide by 1,000” to describe and complete conversions. Avoid shortcuts like “move the decimal point” or “jump the decimal,” as these tricks can lead to misconceptions	

Previously 5.MD.A.1

5.GR.B CLASSIFY TWO-DIMENSIONAL FIGURES BASED ON PROPERTIES.

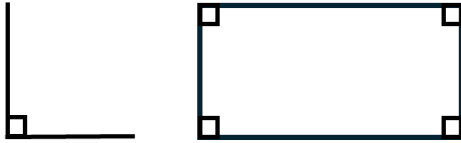
5.GR.B.2: Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category (e.g., all rectangles have four right angles and squares are rectangles, so all squares have four right angles).

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Classify two-dimensional figures based on presence of lines and angles (4.GR.C.9). Where are students going? - Find area of triangles, quadrilaterals, and polygons (6.GR.A.1). - Draw polygons in the Coordinate Plane (6.GR.A.3).	- Attributes are features of a figure that do not change. - Grade 5 students will use many different attributes, including sides (parallel, perpendicular, congruent), angles (type (acute, right, obtuse), measurement, congruent), and symmetry, to describe and categorize two-dimensional figures. - Use notation shown below to note right angles.  - A trapezoid is defined as “A quadrilateral with at least one pair of parallel sides.” Using this definition of a trapezoid, a student may explain that a shape is a trapezoid when it has at least one pair of parallel sides, so a parallelogram is also a trapezoid because it has two pairs of parallel sides.	- Create visuals to show how categories of two-dimensional figures relate to one another (e.g., Venn diagram, tree diagram, etc.). - Practice using precision language (e.g., attribute, polygon, congruent, parallel, perpendicular, right angles, acute angles, obtuse angles, etc.) in their verbal and written explanations (SMP 6). - Construct coherent chains of reasoning that connect attributes of two-dimensional figures to support or refute statements. For example, a student is able to provide evidence to support the claim, “a parallelogram is always a quadrilateral.”

Previously 5.G.B.3

5.GR.B.3: Classify two-dimensional figures based on properties.

- a. Classify triangles based on properties (e.g., angle measure, side lengths).
- b. Classify quadrilaterals in a hierarchy based on properties (e.g., side lengths, angle measures, presence of parallel or perpendicular sides).

Content Progression	Examples and Clarifications	Students should have the opportunity to...						
Where are students coming from? - Explain that attributes of a figures in the same category are shared by all of its subcategories (5.GR.B.2). Where are students going? - Solve problems in context involving area, surface area, and volume (6.GR.A).	- Grade 5 students will use properties, including sides (parallel, perpendicular, congruent) and angles (type, measurement, congruent) to classify triangles and quadrilaterals. - Use notation shown below to note right angles.  - A trapezoid is defined as “A quadrilateral with at least one pair of parallel sides.”	- Explore of variety of sets of two-dimensional shapes in order to classify the shapes. - Use diagrams to classify two-dimensional shapes based on their properties. For example, a student may use a chart to classify two-dimensional shapes based on the presence of congruent sides, as shown below. <table border="1"> <thead> <tr> <th>All sides congruent</th><th>Some sides congruent</th><th>No sides congruent</th></tr> </thead> <tbody> <tr> <td>  </td><td>  </td><td>  </td></tr> </tbody> </table> - Practice using precise language (e.g., attribute, polygon, congruent, parallel, perpendicular, right angles, acute angles, obtuse angles, etc.) in their verbal and written explanations (SMP 6). - Use examples and counterexamples to support or refute mathematical statements or claims. For example, a student may provide a counterexample to refute the statement that to be classified as a right triangle, 2 sides of the triangle will be congruent. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Geometric Reasoning and Measurement milestones in the Classifying Figures strand of the Numeracy Development Framework.	All sides congruent	Some sides congruent	No sides congruent			
All sides congruent	Some sides congruent	No sides congruent						
								

Previously 5.GR.B.4

5.GR.C UNDERSTAND CONCEPTS OF VOLUME AND RELATE VOLUME TO MULTIPLICATION.

5.GR.C.4: Identify volume as an attribute of three-dimensional figures and measure volume by counting cubic units (e.g., non-standard equal-sized units, cubic centimeters, cubic inches, and cubic feet) that fill a figure without gaps or overlaps.

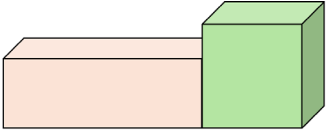
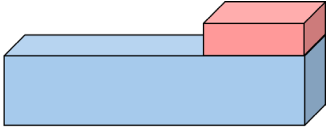
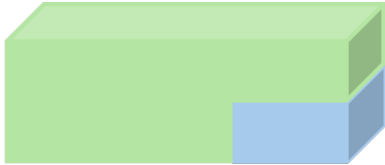
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Apply the area and perimeter formulas for rectangles in context (4.GR.A.3). Where are students going? - Relate volume to the operations of multiplication and addition and solve problems in context (5.GR.C.5). - Find the volume of a right rectangular prism with fractional edge lengths (6.GR.A.2).	- Volume is the measure of the total number of cubic units needed to fill a three-dimensional figure and are measured in cubic units. - Volume of a rectangular prism can be found by counting cubic units or by multiplying the dimensions. This standard is focused on filling three-dimensional figures and counting cubic units. - Grade 5 students will work with whole number edge lengths only. - Provide examples to support students in recognizing the difference between linear, square, and cubic units. Linear units are one-dimensional and measure the length or height of an object. Square units are two-dimensional and measure space. Cubic units are three-dimensional and measure volume. Discuss the difference between these types of units. linear unit — square unit — cubic unit A cube with side length 1 unit, called a “unit cube,” and measures “one cubic unit” of volume - When using cubic units to measure volume, students will fill the figure with equal-sized cubic units (e.g., unit cubes) without gaps or overlaps. Then, count the number of cubic units to determine the volume. The volume of the rectangular prism is 8 cubic units. - Present volume tasks in authentic contexts (e.g., packing boxes).	- Engage in hands-on experiences to develop an understanding of volume. Students should use unit cubes to fill three-dimensional figures and count cubic units to identify the volume. - Count by ones or skip count when counting cubic units to measure volume. Students should begin counting by ones. As they become more proficient, students can transition to skip counting cubic units as a more efficient strategy. - Practice using appropriate units when describing and recording volume (e.g., cubic centimeters, cubic inches, cubic units). - Discuss and explain the difference between area and volume. Make connections to everyday situations or objects that would be described or measured using area versus volume (e.g., an art teacher would measure the area of a painting but measure the volume of a case of clay). Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Geometric Reasoning and Measurement milestones in the Reasoning with Geometric Structure strand of the Numeracy Development Framework.

Previously 5.MD.C.3; 5.MD.C.4

5.GR.C.5: Relate volume to the operations of multiplication and addition and solve problems in context.

- Find the volume of a right rectangular prism with whole-number side lengths by filling it with unit cubes in layers and show that the volume is the same as would be found by multiplying the edge lengths, or by multiplying the height by the area of the base. Represent products of three whole numbers as volumes, (e.g., to represent the associative property of multiplication).
- Apply the formulas $V = (l)(w)(h)$ and $V = (B)(h)$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve problems in the context.
- Determine the volumes of composite figures by decomposing them into non-overlapping right rectangular prisms and adding their volumes together to solve problems in context.

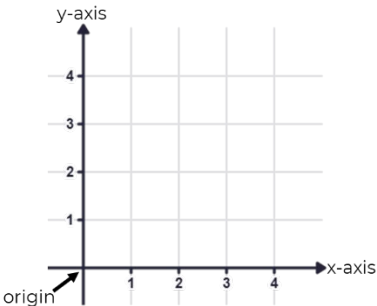
Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Apply the area and perimeter formulas for rectangles in context (4.GR.A.3). - Measure volume by counting cubic units (5.GR.C.4). Where are students going? - Find the volume of a right rectangular prism with fractional edge lengths (6.GR.A.2).	- Volume is the measure of the total number of cubic units needed to fill a three-dimensional figure and are measured in cubic units. - Volume of a rectangular prism can be found by filling a figure and counting cubic units or by multiplying the dimensions. - Grade 5 students will work with whole number edge lengths only. 5.GR.C.5a: Facilitate exploration to allow students to make connections between filling and multiplication. Provide multiple examples to support students in making the generalization. As students begin to generalize, they apply multiplication to calculate volume of rectangular prisms without filling or counting cubic units. The base of the figure is made up of 2 rows of 4 cubes. There are a total of 3 layers. $2 \times 4 \times 3$ 5.GR.C.5b: As students notice patterns, encourage students to describe the relationship using mathematical language and symbols. Allow students to derive and test the formulas, $V = (l)(w)(h)$ and $V = (B)(h)$. Avoid giving students the formulas without student exploration and discovery. <i>Continued on next page</i>	- Engage in filling of various rectangular prisms to identify the relationship between volume, multiplication, and addition. - Use visual and symbolic representations to explain how to determine the volume of a rectangular prism. - Develop their proficiency in calculating volume by initially filling rectangular prism to calculate volume. As students become more proficient, they will transition to using multiplication to calculate volumes without needing visual cubic units. - Use appropriate units when describing and recording volume (e.g., cubic centimeters, cubic inches, cubic units). - Flexibly decompose composite figures in different ways to calculate partial and total volumes. Encourage students to discuss and compare their results. - Use visual and symbolic representations to explain how to determine the volumes of a composite figure. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Geometric Reasoning and Measurement milestones in the Reasoning with Geometric Structure strand of the Numeracy Development Framework.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	5.GR.C.5c: A composite figure is a figure made up of two more rectangular prisms joined together and sharing edges. 5.GR.C.5c: Composite figures can be decomposed in multiple ways to calculate volume. In the example below, the student decomposed the figure into two different rectangular prisms and added each of the smaller volumes to find the total volume.  $(2 \times 6 \times 2) + (2 \times 3 \times 3) = 42$ cubic units  $(2 \times 9 \times 2) + (2 \times 3 \times 1) = 42$ cubic units 5.GR.C.5c: Students may notice that removing part of a rectangular prism can create a composite figure. For example, if the $2 \times 2 \times 1$ prism (blue) is removed from the $2 \times 6 \times 2$ prism, the composite figure is formed (green). To find the volume, calculate the volume of the large prism ($2 \times 6 \times 2$) and subtract the volume of the removed prism ($2 \times 2 \times 1$), resulting in 20 cubic units.  $(2 \times 6 \times 2) - (2 \times 2 \times 1) = 20$ cubic units - Present volume tasks in authentic contexts (e.g., packing boxes). Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students' everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved.	

Previously 5.MD.C.5

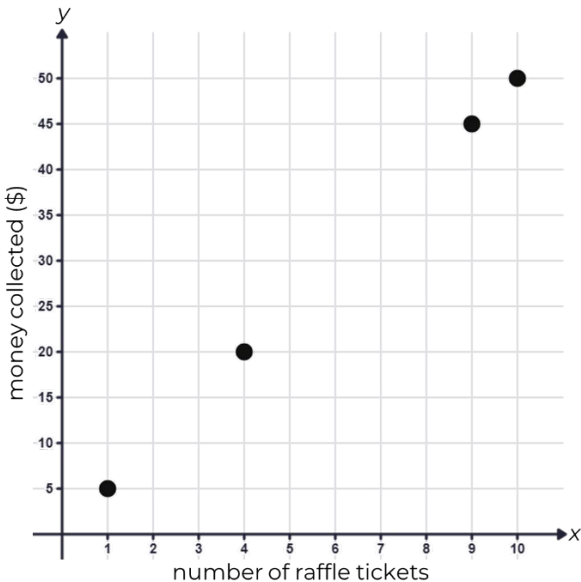
5.GR.D GRAPH POINTS ON A COORDINATE PLANE TO SOLVE CONTEXTUAL AND MATHEMATICAL PROBLEMS.

5.GR.D.6: Define the coordinate system as a pair of perpendicular lines called axes that intersect at the origin. Use understanding that the coordinate values represent the distance from the origin on the x-axis and y-axis to graph and name coordinate points in the first quadrant using ordered pairs.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Represent a fraction as a number on the number line (3.NOS.F.12). Where are students going? - Represent a rational number as a point on vertical or horizontal number lines. Represent points on a number line with negative numbers (6.NOS.C.6).	- Grade 5 students work with positive numbers only, therefore, they will work within the first quadrant of the coordinate plane. - Students should be able to identify key components of a coordinate system, including the origin, the x-axis, and the y-axis.  - Support students in making connection between the number line and the coordinate system. - Provide examples to support students in recognizing the first number in a coordinate pair indicates how far to travel from the origin in the direction of the x-axis, and the second number indicates how far to travel in the direction of the y-axis.	- Physically engage with the coordinate system to explore the structure and develop understanding of how it is used to plot points. Creating a physical coordinate system on the classroom floor allows students to interact with coordinate system and move along the axis to plot coordinates. - Practice using precise language (e.g., origin, x-axis, y-axis, coordinate, ordering pairs, etc.) in their verbal and written explanations (SMP 6). - Explain the difference between ordered pairs where the coordinates are switched. For example, a student can explain the difference between (2, 7) and (7, 2). - Apply knowledge of the coordinate system to explain how to graph coordinate points. Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Geometric Reasoning and Measurement milestones in the Spatial Reasoning strand of the Numeracy Development Framework.

Previously 5.G.A.1

5.GR.D.7: Represent problems by graphing points in the first quadrant of the coordinate plane and interpreting coordinate values of points in the context of the situation.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Graph and name coordinate points in the first quadrant using ordered pairs (5.GR.D.6). Where are students going? - Represent a rational number as a point on vertical or horizontal number lines. Represent points on a number line with negative numbers (6.NOS.C.6).	- Grade 5 students work with positive numbers only, therefore, they will work within the first quadrant of the coordinate plane. - Present authentic contexts and allow students to explore how the coordinate plane may be used to represent a situation. - Context refers to the situation or setting that gives meaning to mathematical ideas, quantities, representations, or relationships. A context may arise from students' everyday experiences, other disciplines, or within mathematics itself and provides a basis for making sense of and reasoning about the mathematics involved. The volleyball team is selling raffle tickets as a fundraiser. Each raffle ticket costs \$5. Four of the teams raffle ticket sales are shown on the coordinate plane.  <i>Continued on next page</i>	- Engage with the coordinate system to represent problems in context and interpret coordinate values. - Practice using precise language (e.g., origin, x-axis, y-axis, coordinate, ordering pairs, etc.) in their verbal and written explanations (SMP 6). - Create their own scenarios to be represented on a coordinate plane.

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	A designer is developing a map for a new section of an Amusement Park. Locations of some of the new attractions are shown on the coordinate plane. • Prompt students to interpret the meaning of the coordinate values based on the context. For example, when interpreting coordinate values on the first coordinate plane above, a student may explain that coordinate (9, 45) represents the team member that sold 9 raffle tickets and collected a total of \$45. When interpreting the coordinate values on the second coordinate plan above, a student may explain that coordinate (3, 2) shows that someone would travel from the entrance 3 units horizontally and then 2 units vertically to arrive at the Live Show.	

Previously 5.G.A.2

Reasoning with Data, Statistics, and Probability (DS)

Reasoning with Data and Statistics Clusters	Connections to the Numeracy Development Framework
5.DS.A Represent and interpret data.	Standards within this cluster directly contribute* to the Grade 5 Reasoning with Data, Statistics, and Probability milestones within the Collecting & Representing Data, Interpreting & Analyzing Data, and Reasoning about Probability strands of the Numeracy Development Framework.

***Directly contribute to milestones:** The cluster includes content and practices that are *explicitly named within* an NDF milestone, meaning student proficiency with this cluster demonstrates attainment of that milestone.

****Contribute to milestones:** The cluster builds prerequisite knowledge, skills, or ways of reasoning that *support progress toward* a milestone in the NDF, even if the milestone itself is not fully realized at this stage.

5.DS.A REPRESENT AND INTERPRET DATA.

5.DS.A.1: Ask and answer questions by collecting, organizing and summarizing data, recognizing the importance of context when analyzing data.

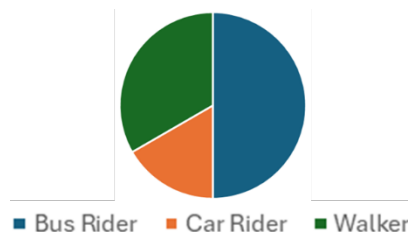
- a. **Create scaled data visualization (e.g. bar graphs for categorical data; line plots with fraction units of halves, fourths, and eighths for numerical data) to display data and communicate relationships or support a claim.**
- b. **Summarize data presented in scaled visualizations (e.g. bar graphs, circle graphs, line plots, line graphs) by identifying the mode, the range, and any gaps in data and draw conclusions.**
- c. **Evaluate whether a data visualization accurately represents the data and allows for accurate interpretation given the context.**

Content Progression	Examples and Clarifications	Students should have the opportunity to...
Where are students coming from? - Ask and answer questions by organizing (bar graphs, line plots), and summarizing data (bar graphs, circle graphs, line plots, line graphs); Compare and contrast data visualizations (4.DS.A.1). Where are students going? - Describe data set by its center, spread, or overall shape (6.DS.A.3). - Interpret numerical data in plots on a number line, including line plots, histograms, and box plots (6.DS.B.5). <i>Continued on next page</i>	- Understanding data goes beyond making graphs. It involves engaging in the full statistical process: asking meaningful questions, collecting and organizing data, and interpreting results to make sense of the world. Authentic applications help students see data as a tool for inquiry and decision-making, not just display. - Categorical data refers to data that can be sorted into groups or categories (e.g., transportation to school or choice for class field trip, etc.). - Consider the reasonableness of the number of categories and/or number of data points when selecting problems and tasks, to ensure data are not unnecessarily cumbersome to display or interpret. 5.DS.A.1a: Students will analyze different data visualizations and/or features of data visualizations to determine best option to communicate a particular relationship or support a claim. For example, a student may choose to create a bar graph with a more concentrated scale to communicate that the difference between the categories is minimal or may use more spread-out scale to communicate the difference between the categories is greater. 5.DS.A.1a: A bar graph represents data using bars and labeled axes. Scales used in bar graphs should be reasonable and allow for manageable skip-counting. When using a scale other than 1, students will use estimation to represent data points that are not multiples of the scale. <i>Continued on next page</i>	- Create their own statistical questions based on their interests and develop a plan to collect data. - Identify appropriate data visualization to use to present a collected or given data set to communicate relationships or support a claim (SMP 5) and explain how their data visualization communicates the relationship or supports the claim. - Determine what scale would be best to represent data and how a different scale might change how data is interpreted (SMP 5). - Explain what the mode, range, or gaps mean in terms of the context of the data. - Make connections between circle graphs, decimals (5.NOS.A) and fractions (5.NOS.C; 5.NOS.D). - Consider actual data set and context when evaluating accuracy of data visualization and whether it allows for appropriate interpretation of the data or if it is misleading. - Discuss and explain how data visualization accurately represents the data or why it does not. - Interpret data, especially when working with mode and range, to make generalizations and consider what is typical or likely within a data set. This builds foundational reasoning about uncertainty that becomes more explicitly "probability" in later grades. <i>Continued on next page</i>

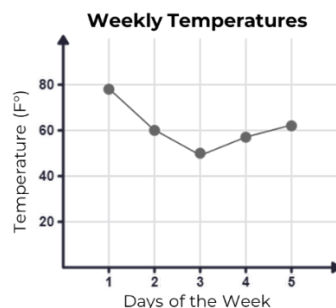
Where are students going?
Summarize numerical data sets in relation to their context (6.DS.B.6).

- **5.DS.A.1a:** A line plot represents data using symbols on a number line. Students will use whole numbers and fractions including halves, fourths, and eighths.
- **5.DS.A.1b:** A circle graph represents data using fractions of a circle. The entire circle represents the whole and different fractions of the circle represent the fraction of total responses for a given category. While circle graphs are often connected to percentages, Grade 5 students do not work with percents so the focus should be on making connections to fractions. Note, students are expected to summarize data presented in a circle graph. They are not expected to create circle graphs as part of this standard.

Ways Students Get to Schools



- **5.DS.A.1b:** A line graph represents the change in data over time. Line graphs connect data points with straight lines. Note, students are expected to summarize data presented in a line graph. They are not expected to create line graphs as part of this standard.



- **Students must have multiple opportunities and varied ways to demonstrate mastery, as this standard constitutes key Grade 5 Reasoning with Data, Statistics, and Probability milestones in the Collecting & Representing Data, Interpreting & Analyzing Data, and Reasoning about Probability strands of the Numeracy Development Framework.**

Content Progression	Examples and Clarifications	Students should have the opportunity to...
	Continued on next page	
	5.DS.A.1b: Introduce mode, range, and gaps as measures that can be used when summarizing data. Mode identifies the value in the data set that appears most frequently. For example, in the Monthly Rainfall data set, the mode is $3\frac{1}{2}$ inches. Range identifies the difference between the greatest and the least values in the data set. For example, in the Monthly Rainfall data set, the range is $1\frac{6}{8}$ inches. Gap identifies large spaces within the data set where data points do not exist. For example, in the Monthly Rainfall data set, a gap exists from $2\frac{1}{8}$ to $2\frac{6}{8}$ inches. Monthly Rainfall (in inches) Rainfall (inches) 5.DS.A.1c: Encourage students to analyze data visualization alongside the data presented in table. This will allow students to recognize whether the graph accurately represents the data set or if the graph presents that data in a way that may be misleading.	

Previously 5.MD.B.2

References

Hulbert, E. T., Petit, M. M., Ebby, C. B., Cunningham, E. P., & Laird, R. E. (2023). *A focus on multiplication and division: Bringing mathematics education research to the classroom* (2nd ed.). Routledge.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2020). *Elementary and middle school mathematics: Teaching developmentally* (10th ed.). Pearson

Some graphics obtained from <https://www.flaticon.com>

Provide Feedback

Your feedback is essential in helping us understand how these resources are being used, their effectiveness in supporting instruction, and how they can be improved to better serve educators, students, and stakeholders across the state. If you are interested in providing feedback, please completed the [Feedback: PreK-8 Mathematics Standards Companion Guide](#) survey.

The survey includes general questions about your overall impressions of the companion guides, as well as opportunities to provide grade-level specific feedback. Your responses will inform future revisions and professional learning efforts to ensure the guides are clear, useful, and aligned with the needs of Maryland's diverse learning communities.